AUTHOR

B. Harinath Reddy
B.Tech., M.B.A

Jr. Lecturer in Civil Engg.
Govt. Jr. College (New)
Malakpet, Hyderabad.

EDITOR

A.Srinivasa Reddy
M.Tech, (T.E.)

Jr. Lecturer in Civil Engg.
Govt. Jr. College,
Aliya, Hyderabad.

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Intermediate
Vocational Course
First Year

Engineering Mechanics
For the Course of Construction Technology

Board of Intermediate Education
Govt. of Andhra Pradesh
Hyderbad.
2007

CONTENTS

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1
SYSTEMS OF MEASUREMENTS

1.1 Introduction

Engineering Mechanics is the branch of science, which deals with the study of Laws of mechanics and the application of principles of mechanics to solve Engineering problems. The study of Engineering Mechanics is important for an engineer in planning, design and construction of structures and machines. Mechanics deals with study of Laws of force and its effect on bodies. Mechanics may be divided into statics and Dynamics.

Statics is that branch of mechanics which deals with the study of the force and their effect on bodies at rest. Dynamics is that branch of mechanics which deals with the study of bodies which are in motion. Dynamics is again subdivided into kinematics and kinetics. Kinematics is the study of pure motion whereas kinetics deals with both the cause and effect of motion.

1.2 Base units and Derived units:

The measurements of physical quantities is one of the most important operations in engineering. The units of physical quantities which are used to express units of other physical quantites are called as Base or fundamental units, Length , mass and time are fundamental quantities and their units are called fundamental units.

The units of other physical quantities which can be expressed in terms of fundamental units are called derived units. For example, the units of area, volulme, speed, etc., are derived units (Table1.2)

1.3 Systems of Measurements

There are four systems of measurements which are commonly used. They are

1. FPS System 2. C G S System 3. MKS System
4. SI System

1. FPS System :

In this system the units of fundamental quantities length, mass and time are foot, pound and second respectively.

2. CGS Sytem :

In this system the units of fundamental quantities length, mass and time are centimetre, gram and second respectively.

3. MKS System :

In this system the units of fundamental quantities length, mass and time are metre, kilogram and second respectively.

4. S.I. System :

It is noticed that only length, mass and time are not sufficient to derive physical quantities conveniently. In addition to these, four more physical quantities and two supplementary quantities are taken. This system is called International System of Units(SI) and was agreed at the Eleventh International Conference of Weights and Measures in 1960.

TABLE 1.1 SI UNITS

Sl.No	Physical Quantity	Unit	Unit Symbol
	Base Units		
1.	Length	metre	m
2.	Mass	kilogram	kg

3.	Time	Second	S
4.	Electric current	Ampere	A
5.	Thermodynamic	Temperature Kelvin	K
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol
	Supplementary Units		
1.	Plane angle	radian	rad
2.	Solid angle	steradian	sr

Table 1.2 Derived Units

Sl.No	Quantity	Unit	Symbol
1.	Area	Square metre	M^2
2.	Volume	cubic metre	M^3
3.	Velocity	metre/ second	m/s
4.	acceleration	metre/sq. second	m/s^2
5.	Radius of gyration	metre	m
6.	Density	Kilogram-cubicmetre	$Kg\text{-}m^3$
7.	Pressure	Newton/sq.metre	N/m^2
8.	Stress	newton/sq.metre	N/m^2

1.4 Multiples and Sub-Multiples

The International System (SI) of units recommends the following multiples and sub multiples for prefixes to be used.

tera (T)	10^{12}	giga(G)	10^9	mega (M)	10^6
Kilo(K)	10^3	milli(m)	10^{-3}	micro(μ)	10^{-6}
nano(n)	10^9	pico(P)	10^{-12}	femto(f)	10^{-15}

1.5 Definition of units of SI System

1. Metre: It is the length equal to 1,650,763.73 times the wave length in a vaccum of the radiation of orange-red light of Kripton-86 atom.

2. Kilogram: It is the mass of the Platinum-Iridium Cylinder kept at the International Bureau of weights and measures at Serves (Paris).

3. Second: It is the time equal to the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of Ceasium-133atom.

4. Kelvin: It is the fraction 1/273.16 of the thermodynamic temperature of the friple point of water. It is the unit of thermo dynamic temperature.

5. Ampere : It is an electrical current produced between two parallel conductors placed at a distance of one metre length in vaccume. It produces a force of 2×10^{-7} Newtons per metre of the length.

6. Candela : It is defined as one sixtieth part of luminous intensity emitted by one square centimetre of a black body radiator at the temperature of solidification of platinum.

7. Mole: It is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012Kg of carbon 12.

Short Answer Questions

1. Define engineering Mechanics.
2. Explain the branches of mechanics.
3. Define base and derived units.
4. What are the systems of measurements.
5. State the physical quantities with units in SI System.

2
FORCES AND MOMENTS

2.1 Force

Newton's first law of motion states that, 'Everybody continues in its state of rest or of uniform motion unless it is compelled by an external force to change that state". Hence, force is an action which changes or tends to change the state of rest of a body or of its uniform motion.

For example, a ball on a table moves when it is pushed. A moving car stops when brakes are applied. Here, the force changes the state of rest of ball and changes the motion of the car.

2.2 Effects of a force

The following effects may produce in a body when force is acting on it:

a) If the body is at rest, the force may set it into motion.

b) If the body is in motion, the force may accelerate or retard the motion.

c) It may balance the force, already acting on a body bringing the body to rest or in equilibrium

d) It may develop internal stresses in the body, on which it acts.

2.3 Characteristics of a force

The following are the characteristics of a force:

a) Magnitude of the force (Ex 10N, 20 N, etc.,)

b) Direction of force i.e., time of action of force.

c) Nature of the force i.e. push or pull.

d) The point through which the force acts on a body.

2.4. Vectors and scalars

Most of the physical quantities measured in science are classified as either vectors or scalars.

Vector: A physical quantity which has magnitude and direction is called as vector.

Example: Velocity, momentum, force etc.

Scalar : A physical quantity which has only magnitude but no direction is called as scalar.

Example: Mass, time, temperature, density, etc.,

2.5 Unit of force

The unit of force in SI System is Newton. Newton is the force which, when applied to a body having a mass of 1 kg gives it an acceleration of 1m /s^2.

2.6 Systems of forces

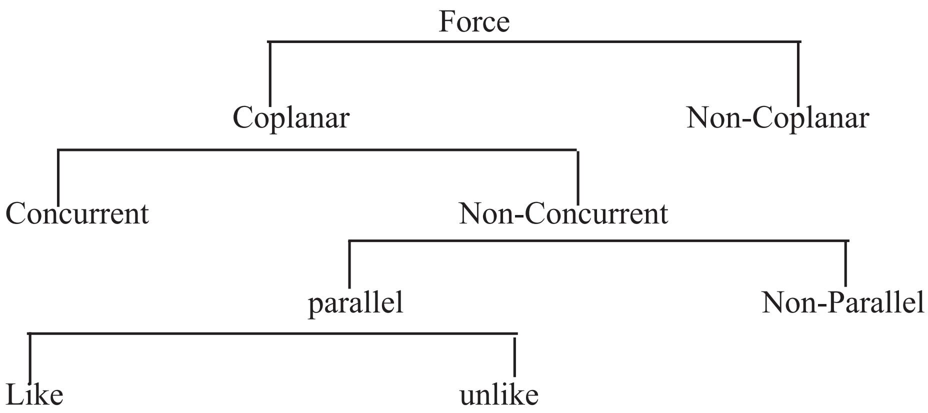


Fig 2.1 Systems of Forces

Forces can be broadly classified into

a) Coplanar Forces: The forces, whose lines of action lie on the same plane, are called as coplanar forces (Fig.2.2)

b) Non-Coplanar Forces: The forces, whose lines of action do not lie on the same plane are called as non-coplanar forces. (Fig.2.3)

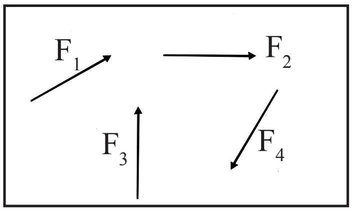


Fig.2.2

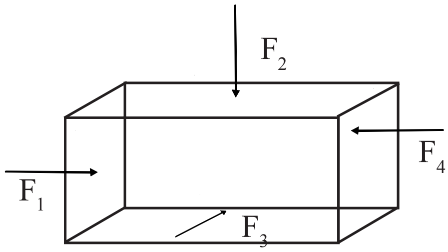


Fig.2.3

c) Coplanar concurrent forces: The forces, whose lines of action lie in the same plane and meet at one point, are called as coplanar concurrent forces (Sec Fig.2.4)

d) Coplanar non concurrent forces: The forces, whose lines of action lie in the same plane, but do not meet at one point are called as coplanar non-concurrent forces.(Fig.2.5)

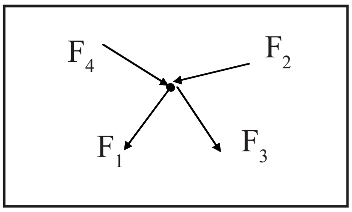


Fig.2.4

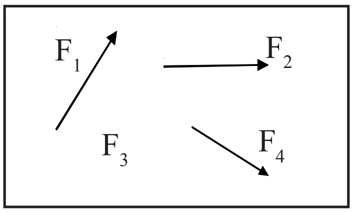


Fig.2.5

e) Coplanar non-concurrent parallel forces: The forces, whose line of action lie in the same plane, do not meet at one point but parallel to each other are called as coplanar non-concurrent parallel forces (fig 2.6)

f) Coplanar non-concurrent non parallel forces: The forces, whose line of action lie in the same plane, do not meet at one point and not parallel to each other are called as coplanar non-concurrent non par allel forces.

g) Coplanar non-concurrent like parallel forces: The forces, whose line of action lie in the same plane, do not meet at one point parallel to each other and acting in the same direction are called as coplanar non-concurrent like parallel fores(fig2.7)

h) Coplanar non-concurrent unlike parallel forces: The forces, whose line of action lie in the same plane, do not meet at one point, paral

-lel to each other and do not act in the same direction are called as coplanar non-concurrent unlike forces.(Fig.2.8)

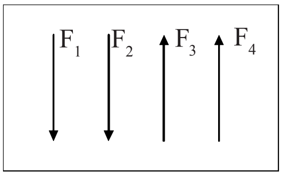


Fig.2.6

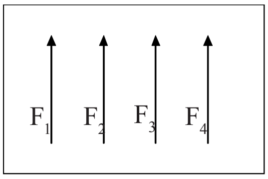


Fig.2.7

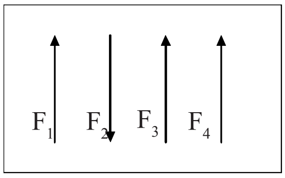


Fig.2.8

2.7. Definition of Terms

a) Representation of a force: Force is a vector quantity. Magnitude, direc -tion and point of application of a force are required to represent it (Fig.2.9)

b) **Resultant force:** When a system of forces acting on a body are re placed by a single force which produces the same effect of all the forces put together is called as resultant force. Let F_1, F_2,F_3, etc., are the forces acting on a body. R is the resultant of all the forces which cause the same effect. The body has a tendency to move in the direction of the resultant force.(Fig.2.10).

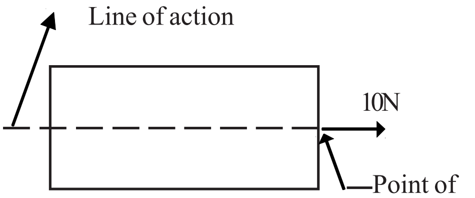


Fig. 2.9

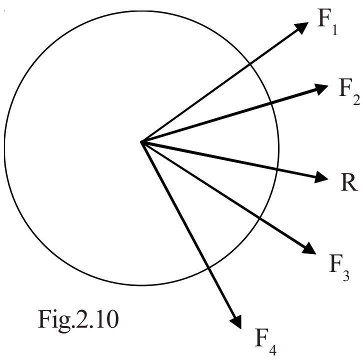


Fig.2.10

c) **Equilibrium:**

When the resultant of a system of forces acting on a body is zero, the body will be in equilibrium. The force, which applied to bring the body into rest state is called equilibrant and it is equal in magnitude and opposite in direction of the resultant force(fig 2.11)

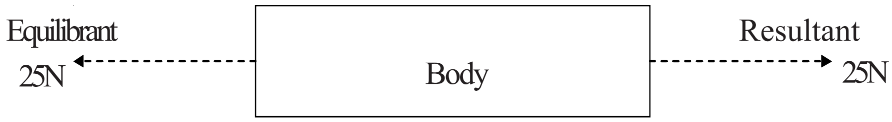


Fig. 2.11

d) Resultant of collinear forces:

Forces acting in a straight line are called as collinear forces. The resultant of collinear forces is the algebraic sum of forces, taking the forces acting in one direction as positive and the other direcion as neagative.(fig.2.12).

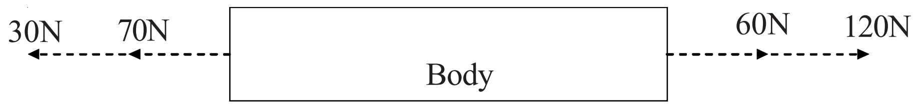


Fig. 2.12

Sum of forces to the right = 60+120 = 180N

Sum of forces to the left = 70 + 30 = 100 N

Resultant = 180 - 100 = 80 N to the right.

2.8. Methods of finding resultant of forces

The resultant force of a given system of forces may be found out by the following methods.

1) Law of parallelogram of forces
2) Method of resolution
3) Graphical methods

1) Law of parallelogram of forces

The Law states that "If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelo -gram, their resultant will be represented in magnitude and direction by the diagonal of the parallelogram, which passes through the point of intersection".

Let two forces F_1 and F_2 acting at O, be reprsented by the straight lines OA and OC in magnitude and direction as shown in fig 2.13. Com plete the parallelogram OABC. Join OC, which is resultant force. Let θ is the angle between two forces and α is the angle between resultant and force F_1. Produce the line OA and draw a perpendicular from B to meet OA at M.

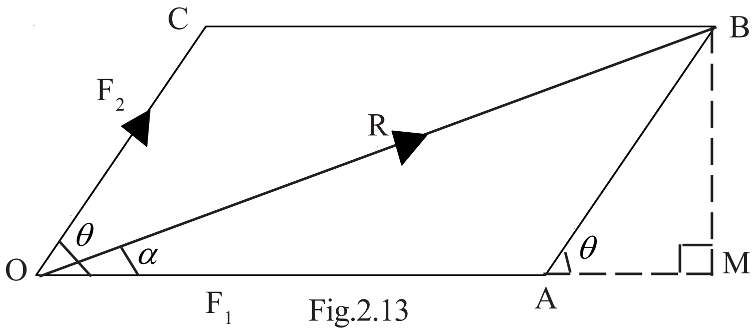


Fig.2.13

$in \Delta ABM$

$$AB = F_2, \angle BAM = \theta$$

$$Sin\theta = \frac{BM}{AB}, Cos\theta = \frac{AM}{AB}$$

$$\therefore BM = F_2\, Sin\theta \; and \; AM = F_2\, Cos\theta$$

$in \Delta OBM,$ $\quad OB^2 = OM^2 + BM^2 = (OA + AM)^2 + BM^2$

$$OB^2 = (F_1 + F_2\, Cos\theta)^2 + (F_2\, Sin\theta)^2$$

$$\therefore \text{Resultant}(R) = \sqrt{F_1^2 + F_2^2\, Cos^2\theta + 2F_1 F_2\, Cos\theta + F_2^2\, Sin^2\theta}$$

$$= \sqrt{F_1^2 + F_2^2(Cos^2\theta + Sin^2\theta) + 2F_1F_2\, Cos\theta}$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\, Cos\theta} \quad (\because Sin^2\theta + Cos^2\theta = 1)$$

Let the resultant (R) is making an angle α with OA,

$$\tan\alpha = \frac{BM}{OM} = \frac{F_2 Sin\theta}{F_1 + F_2\, Cos\theta}$$

Case : (1) If θ=0, i.e.,when the forces acting in the same line, $R=F_1+F_2$

2) If $\theta=90^0$, i.e., when the forces acting at right angles, $R = \sqrt{F_1^2 + F_2^2}$

Example 2.1

Find the resultant of the two forces whose magnitudes are 20KN and 50 KN acting an an angle of 60^0 with each other, using Law of parallelogram of forces.

Solution:

Two forces F_1 = 20KN and F_2 = 50 KN
Angle between the forces = $\theta = 60^0$

$$\therefore \text{Resultant of two forces,} \quad R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

$$= \sqrt{20^2 + 50^2 + 2 \times 20 \times 50 \times \cos 60}$$

$$= \sqrt{400 + 2500 + 1000} = \sqrt{3900} = 62.45KN$$

α is the angle between resultant and force F_1

$$\text{Tan}\,\alpha = \frac{F_1 \sin\theta}{F_1 + F_2 \cos\theta} = \frac{50 \sin 60}{20 + 50 \cos 60} = \frac{43.30}{20 + 25} = 0.962$$

$$\alpha = \text{Tan}^{-1}(0.962) = 43.89^0$$

Example 2.2

The resultant of two forces P and Q is 120N. If P is 60N and makes an angle of 30^0 with Q. Determine the magnitude of Q and also direction of resultant. (I.P.E. March 98)

Solution:

Let the two forces are P and Q.
Resultant force (R) = 120 N
Force P = 60N and $\theta = 30^0$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\,Cos\,\theta}$$

$$120\,N = \sqrt{60^2 + Q^2 + 2 \times 60 \times Q\,Cos\,30^0}$$

Squaring both sides

$$120^2 = 60^2 + Q^2 + 60\sqrt{3}\,Q$$

$$\therefore \quad Q^2 + 60\sqrt{3}\,Q + 3600 - 14400 = 0$$

$$Q = \frac{-60\sqrt{3} + \sqrt{\left(60\sqrt{3}\right)^2 - 4 \times 1 \times (-10800)}}{2 \times 1}$$

$$Q = \frac{-103.92 \pm 232.37}{2} = 64.22\,N$$

Let α is the angle between resultant (R) and force P.

$$Tan\,\alpha = \frac{Q\,Sin\,\theta}{P + Q\,Cos\,\theta} = \frac{64.22\,Sin\,30}{60 + Cos\,30} = \frac{64.22 \times 0.5}{60 + 0.866} = 0.53$$

Example 2.3

Find the magnitudes of two forces, such that if they act at right angle their resultant is $\sqrt{113N}$ but if they act at 60^0, their resultant is 13N.

Solution:

Let the two forces are F_1 and F_2.

1) When the angle between the forces $\theta = 90^0$

$$R = \sqrt{F_1^2 + F_2^2}$$

$$\sqrt{113} = \sqrt{F_1^2 + F_2^2}$$

$$\textit{Squaring both sides, } F_1^2 + F_2^2 = 113 \qquad -(1)$$

2) When the angle between the forces $\theta = 60^0$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2\,Cos\theta}$$

$$13 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\,Cos\,60}$$

Squaring both sides and substituting Eq. (1)

$$169 = 113 + 2F_1 F_2 \times \frac{1}{2} \qquad \left(\because Cos60 = \frac{1}{2}\right)$$

$$\therefore \quad F_1F_2 = 56 \qquad -(2)$$

$$Now_1 \quad (F_1 + F_2)^2 = F_1^{\;2} + F_2^{\;2} + 2F_1F_2 = 113 + 2\times 56 = 225$$

$$F_1 + F_2 = \sqrt{225} \qquad = 15 \qquad -(3)$$

$$(F_1 - F_2)^2 = F_1^{\;2} + F_2^{\;2} - 2F_1F_2 = 113 - 2\times 56 = 1$$

$$F_1 - F_2 = \sqrt{1} \qquad = 1 \qquad -(4)$$

Solving the equations (3) *and* (4),

$$F_1 = 8N; \qquad F_2 = 7N$$

Exercise 2.1

1. Two forces acting away from a point whose magnitudes are 150N and 300N making an angle of 60^0 with each other. Find the result ant of forces using Parallelogram Law of forces (I.P.E., March 96)
2. Two forces 80N and 120N are acting at a point 0, at an angle of 90^0 to each other. Find the magnitude and direction of the resultant.
3. The resultant of two forces is 120N. If first force is 50N and makes an angle of 75^0 with second force. Find the magnitude of second force and direction of the resultant.
4. Two forces act at an angle of 150^0. The greater force is 160N and the resultant is perpendicular to the smaller force. Find the smaller force.
5. The resultant of two forces 60N and 100N. If 100N force is doubled, the new resultant is perpendicular to force 60N. Find the magnitude and direction of the resultant.
6. Find the angle between two equal forces F, when their resultant is (i) equal to F, and (ii)equal F/2.

2) Method of resolution :

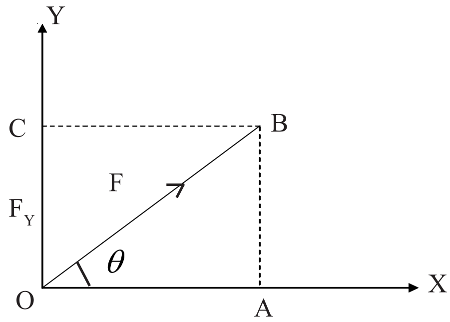
Resolution of a force is defined as dividing the force into a number of components. Generally, the force is resolved into two mutually perpendicular directions.

Consider a force F, is acting at a point O, making an angle θ with X - axis as shown in Fig 2.14

F_X = Resolved part of force F along X - axis

= F cos θ

F_Y = Resolved part of force F along Y - axis
= F sin θ



Consdier system of forces are acting at a point, resolve all the forces horizontally and find the algebraic sum of all horizontal components $\left(\sum F_X\right)$, resolve all the forces vertically and find the algebraic sum of all vertical components $\left(\sum F_Y\right)$. Now the resultant R of the system of forces is given by the equation $R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2}$ and the resultant force is inclined at an angle θ, with the horizontal, then

$$\tan\theta = \frac{\Sigma F_Y}{\Sigma F_X}$$

Note: The value of θ depends on the values of ΣF_X *and* ΣF_Y.

i) When ΣF_X is +ve, θ will be in between 0^0 to 90^0 and 270^0 to 360^0. When ΣF_X is -ve, θ will be in between 90^0 to 270^0.

ii) When ΣF_Y is +ve, θ will be in between 0^0 to 180^0 . when ΣF_Y is -ve, θ will be in between 180^0 to 360^0

Example 2.4

Find the magnitude and direction of the resultant of the four forces 8KN, 10KN, 20KN and 2KN as shown in fig.2.15

Solution:

Resolving all forces along X - axis and y - axis.

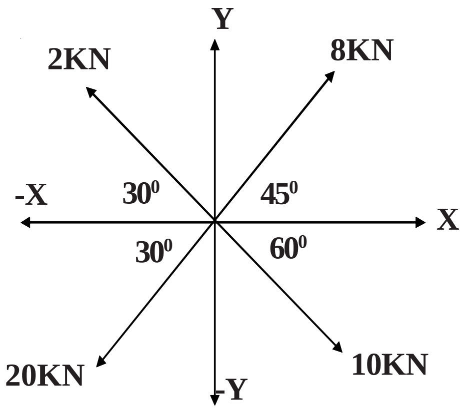


Fig.2.15

Force	**angle**	**F_X = F Cos θ**	**F_Y = F Sin θ**
8KN	45^0	8 Cos 45^0 = 5.656KN	8 sin 45^0 = 5.656 KN
10KN	60^0	10 Cos 60^0 = 5 KN	10 Sin 60 = -8.660KN
20 KN	30^0	-20 Cos 30 = -17.320 KN	-20 Sin 30 = -10 KN
2 KN	30^0	-2 Cos 30 = -1.732 KN	-2 Sin 30= 1 KN
	Total	ΣF_X = -8.396 KN	ΣF_Y = - 12.004 KN

Resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_Y)^2} = \sqrt{(-8.936)^2 + (-12.004)^2}$

$= 14.64 KN$

$$\tan\theta = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-12.004}{-8.396} \Rightarrow \theta = 55^0\, 1^1$$

Example 2.5 A system of forces 2, 3,4,5 and 6 N are acting on one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the direction and magnitude of the resultant.

Solution: **Resolving all forces along X - axis and Y- axis.**

Force	**angle**	**F_X = F Cos θ**	**F_Y = F Sin θ**
2	0^0	2 Cos 0 = 2 N	2 Sin 0^0 = 0
3	30^0	3 Cos 30 = $1.5\sqrt{3}N$	3 Sin 30^0 = 1.5 N
4	60^0	4 Cos 60 = 2 N	4 Sin 60^0 = $2\sqrt{3}$
5	90^0	5 Cos 90 = 0	5 Sin 90^0 = 5
6	120^0	6 Cos 120 = - 3N	6 Sin 120 = $3\sqrt{3}N$
	Total	$\sum F_X = 3.6N$	$\sum F_Y = 15.16N$

Resultant (R) $= \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{3.6^2 + 15.16^2} = 15.58N$

$$\tan\theta = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{15.16}{3.6} = 4.211$$

$$\theta = \tan^{-1}(4.211) = 76^0\, 39^1$$

Fig.2.16 Shows system of forces acting on the angular points of a regular hexagon.

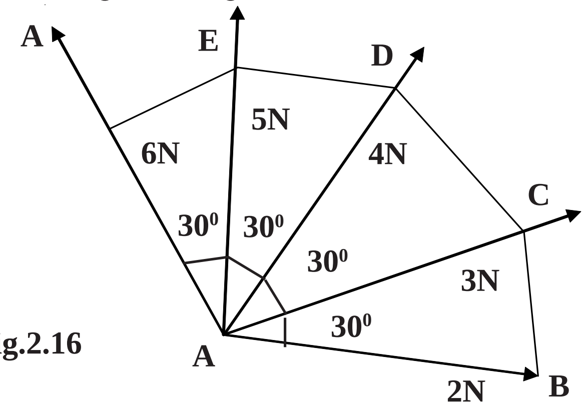


Fig.2.16

Example 2.6 The folowing forces are acting on a particle. Find the magnitude and direction of resultant.

1) 10N indined 30^0 to North of East
2) 15 N towards North
3) 30 N inclined 45^0 North of west
4) 50 N towards south.

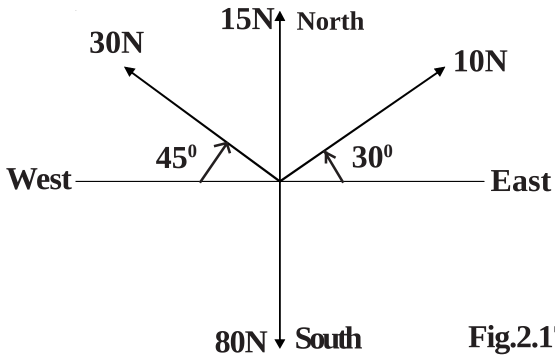


Fig.2.17

Solution : Fig. 2.17 Shows system of forces acting on the particle. Resolving forces along East-West line

$$\Sigma F_x = 10 Cos\ 30 + 15\ Cos 90\ - 30\ Cos\ 45 - 80\ Cos\ 90$$

$$= 5\sqrt{3} = 0\ -\ 21.21\ \ - 0 = -12.55N$$

$$\Sigma F_Y = 10 Sin\ 30 + 15\ Sin\ 90 + 30\ Sin\ 45^0 - 80\ Sin\ 90$$

$$= 5 + 15 + 21.21 \qquad -80 = -38.78N$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{1661.91} = 40.76N$$

$$Tan\ \theta = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{38.78}{12.55} = 3.09$$

$$\theta = Tan^{-1}(3.09) = 72^0\ 04^1 \text{ from East}$$

Example 2.7 A particle which is at centre of a hexagon, acted upon by forces 10N, 35N, 45N, 30N, F_1 and F_2., all in the same plane and radiating from the centre and each force is acting along angular points of the hexagon. Find the forces F_1 and F_2, if the particle is in equilibrium.

Solution : Fig.2.18 Shows the particle 0, which is at Centre of hexagon ABCDEF and acted by system of forces along OA, OB, OC, OD, OE and OF. Resolving the forces along X - axis.

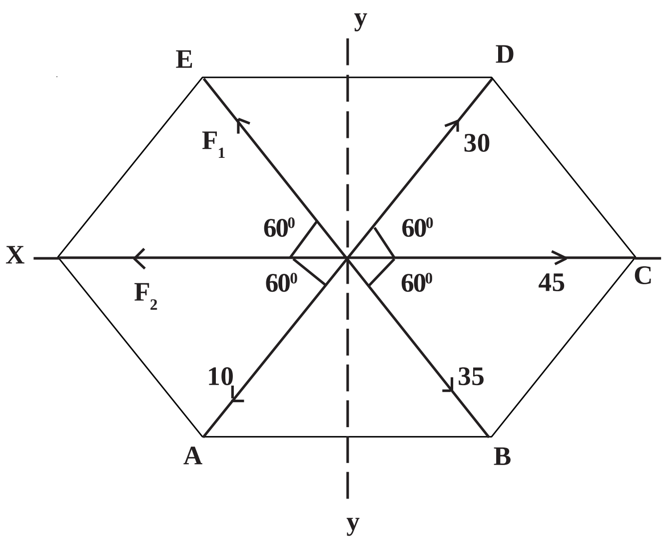


Fig.2.18

$$\Sigma F_X = -10\cos 60^0 + 35\,Cos\,60 + 45\cos 0 + 30\cos 60$$
$$-F_1\cos 60 - F_2\cos 0 = 0$$
$$-5 + 17.5 + 45 + 15 + \frac{-F_1}{2} - F_2 = 0$$
$$72.5\frac{-F_1}{2} - F_2 = 0$$
$$F_1 + 2F_2 = 145 \qquad -(i)$$

Resolving the forces along Y - axis.

$$\Sigma F_Y = -10\sin 60 - 35\sin 60 + 45\sin 0 + 30\sin 60$$
$$+F_1\sin 60 - F_2\sin 0 = 0$$
$$-5\sqrt{3} - 17.5\sqrt{3} + 0 + 15\sqrt{3} + F_1\frac{\sqrt{3}}{2} = 0$$
$$\therefore F_1 = \frac{7.5\sqrt{3}\times 2}{\sqrt{3}} = 15N \quad --(ii)$$

Substitute F_1 Value in equation (i)

$$15 + 2F_2 = 145$$
$$\therefore F_2 = \frac{145-15}{2} = 65N$$

Exercise 2.2

1. Find the magnitude and direction of resultant force, for the system of forces 18N, 12N, 20N and 35N acting at 60^0, 90^0, 150^0, 180^0 respec tively along the horizontal

2. Determine the magnitude and direction of the resultant of forces act ing from a point as given below:
   a) A force of 20N acting due East.
   b) A force 60N acting at 30^0 North of East
   c) A force 100 N acting at 45^0 North of West
   d) A force 80N due South.

   (I.P.E. March 95)

3) Find the resultant of force 10KN, 15 KN, 18KN, 25KN acting at an

angular points of a pentagon, towards the other angular points taken in order.

4) A wheel has 8 spokes equally spaced. The forces acting in 6 con secutive spokes are 50N, 60N, 80N, 100N, 120N, 150N respectively. Find the forces acting in other two spokes

3) Graphical Methods:

The resultant of system of forces may also be found out graphically by the following methods.

a) Law of triangle of forces. b) Law of polygon of forces.

a) Law of triangle of forces:

This law states. " If two forces acting simultaneously on a particle be represented by two sides of triangle taken in order, their resultant is equal to third side of the triangle taken in opposite order.

This can also be stated as "If three forces acting at a point be repre sented in magnitude and direction by the sides of a triangle taken in order, they will be in equilibrium".

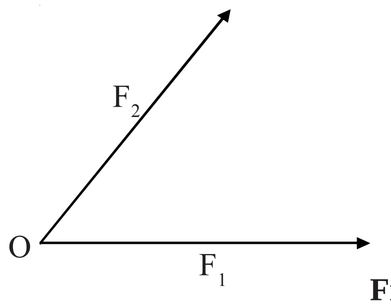


Fig. 2.19

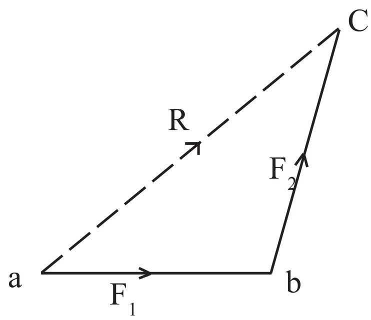


Fig.2.19 Shows two forces F_1 and F_2 acting at point O. Draw a line ab parallel to force F_1 and length equal to magnitude of force F_1. Similarly draw a line bc parallel and equal to force F_2. Join ac, which is the resultant force R in magnitude and direction. The equilibrant is equal and opposite to the resultant.

b) Law of polygon of forces:

This Law states, "If system of forces acting on a particle, be represented in magnitude and direction by the sides of a polygon taken in order, then the resultant of these forces may be represented in magnitude and direction by the closing side of the polygon taken in opposite order.

Let F_1, F_2, F_3 , etc. are the forces acting at a point 'O' as shown in Fig.2.20. Let these forces are represented along the sides of a polygon such that the sides, ab, bc, cd, de and ef, respectively parallel and proportional to forces. Join 'af' which gives the resultant force in magnitude and direction.

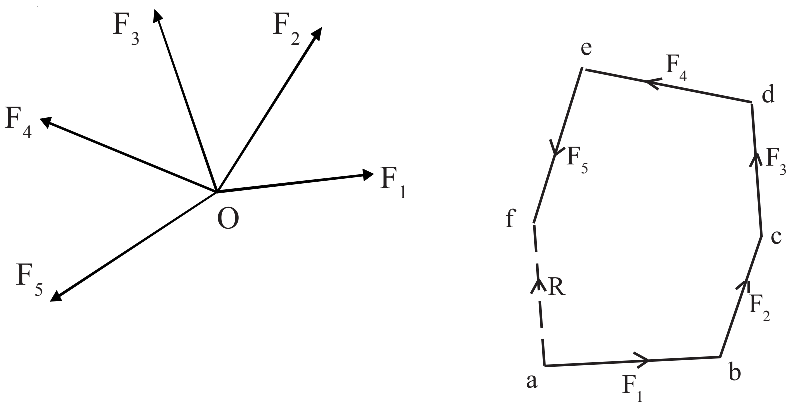


Fig. 2.20

2.9 Lami's Theorem

It states that if three coplanar forces acting at a point be in equilibrium, then each force is poportional to the sine of the angle between the other two.

Let F_1, F_2 and F_3 are three forces acting at a point 'O' as shown in fig 2.21 according to Lami's theorem.

$$\frac{F_1}{Sin\,\alpha}=\frac{F_2}{Sin\,\beta}=\frac{F_3}{Sin\,\gamma}$$

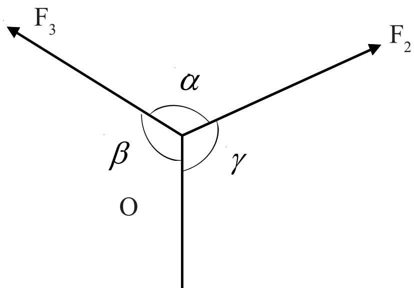


Fig.2.21

Example 2.8 A body of mass 10 KN is suspended by two ropes shown in Fig.2.22. Calculate the forces in the ropes using Lami's Theorem.

Solution.

Let F_1 and F_2 are the forces in ropes AC and BC.
Applying Lamis theorem.

$$\frac{10}{Sin\ 90} = \frac{F_1}{\sin\ 140^0} = \frac{F_2}{Sin\ 130^0}$$

$$\therefore F_1 = \frac{10\ Sin\ 140}{Sin\ 90} = 6.4\,KN$$

$$F_2 = \frac{10\ Sin\ 130}{Sin\ 90} = 7.66\ KN$$

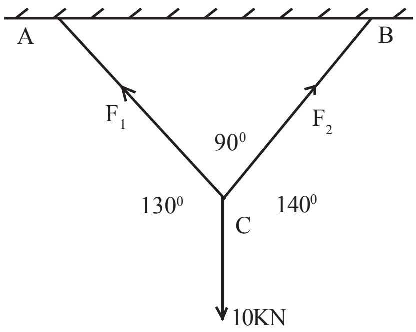


Fig.2.22

Example 2.9 :

A weight of 15 KN hangs from a point as shown in Fig. 2.23 Calculate the forces in strings AC and BC.

Solution :

Let F_1 = force in string AC
F_2 = Force in string BC

From the geometry of the figure

$$\angle ACB = 75^0$$

$$\angle BCO = 135^0$$

$$\angle ACO = 120^0$$

Applying Lami's theorem,

$$\frac{15}{Sin75^0} = \frac{F_1}{Sin135^0} = \frac{F_2}{Sin120^0}$$

$$\therefore F_1 = \frac{15}{Sin75} \times Sin135 = 11KN$$

$$F_2 = \frac{15}{Sin75} \times Sin120 = 13.45KN$$

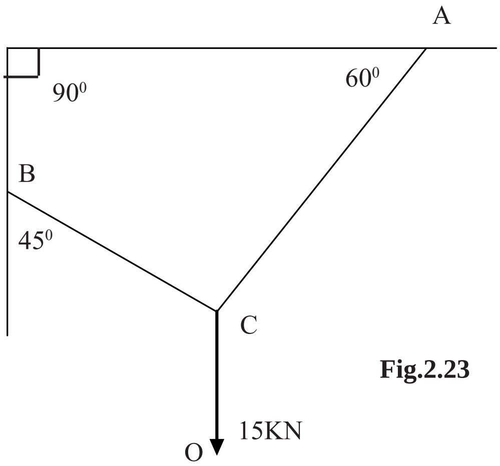


Fig.2.23

2.10. Moment of a force

If a force is acting on a body, it also produces turning effect on the body. It is called as the moment of the force

$$\therefore \quad Moment = F \times l$$

Where F = Force acting on the body

ℓ = The moment arm i.e., the distance between the point and the line of action of force. The moment of force is a vector quantity. The unit for moment is Newton - metre(N-m).

2.11. Types of moments

Moments are mainly classified into two types : (a) clockwise moments (b) Anti - clock wise moments.

The moment of force which turn or rotate the body in clockwise direction is called clockwise moment, as shown in fig. 2.24.

The moment of force which turn or rotate the body in anti - clockwise direction is called anti - clcokwise moment, as shown in fig.2.25

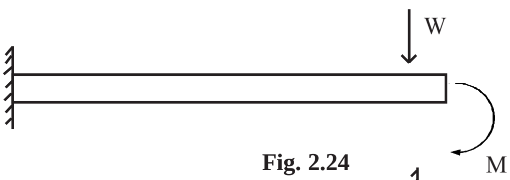


Fig. 2.24

Clock wise Moment

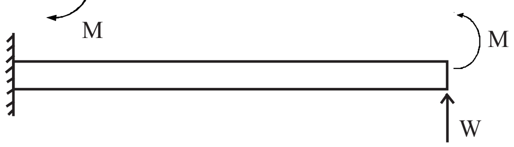


Fig. 2.25 **Anti Clock wise Moment**

2.12. Principle of moments -Varginon's Theorem

Varignon's Theorem states that when a number of forces acting on a body, the algebraic sum of moments about a point is equal to the moment of their resultant about the same point.

Let F_1, F_2, F_3 etc. are the forces acting on a body and the perpendicular distances from a point are l_1, l_2, l_3, etc. Let F is the resultant of all these forces acting at a distance L from the same point, then

$$F \times L = F_1 \times L_1 + F_2 \times L_2 + F_3 \times L_3 + ...$$

$$\therefore \quad M = M_1 + M_2 + M_3 + ...$$

2.13. Law of Moments

It state that "If a system of forces are acting on a body be in equilibrium thus the sum of clockwise moments is equal to sum of anti clockwise moments, about any point. This law is useful in finding the reactions of beams and forces in frames, etc.

The following are the conditions of equilibrium of a rigid body subjected to a system of coplanar forces.

1) The algebraic sum of resolved components of all forces is equal to zero.

$\sum H = 0$ and $\sum V = 0$

$\sum H$ = Resolved components of forces in horizontal direction.

$\sum V$ = Resolved components of forces in vertical direction.

ii) The algebraic sum of moments is equal to zero i.e. $\sum M = 0$

2.14 Couple

Two equal , opposite and parallel forces whose lines of action are different form a couple. The distance between the lines of action of the two forces is called as arm of couple. A couple will not produce motion of a body in a straight line but produces rotation in the body.

2.15. Moment of Couple

The moment of a couple is the product of the force and the arm of the couple.

$$M = F \times d$$

Where M = Moment of couple
F = Force of couple
d = Arm of couple

2.16 Classification of couples

The couples may be, mainly classified into two types.

1) Clockwise couple 2) Anti clockwise couple

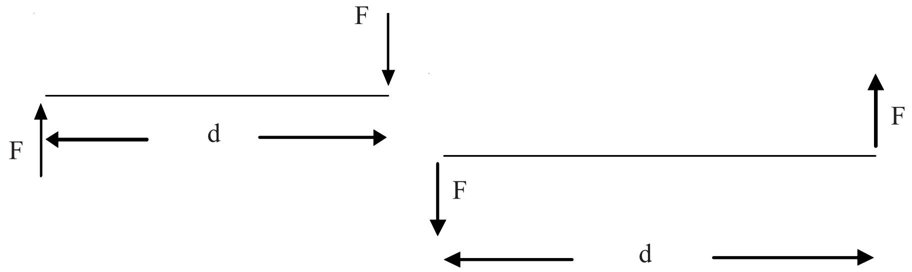


Fig.2.26

A couple, which rotates the body in clockwise direction is called clockwise couple. A couple, which rotates the body in anti-clockwise direction is called anticlockwise couple.

2.17. Characteristics of a couple

A couple (either clockwise or anticlockwise) has the following characteristics.

1) The resultant force of a couple is zero.
2) The algebraic sum of the moments of the forces, forming the couple is constant.
3) A couple can be balanced only by an equal and opposite couple in the same plane.

EXERCISE 2.3

1. A lamp weighing 150N is suspended by two strings of 6m and 8m length fixed to ceiling 10m apart. Find the tension in the strings.

2. Two men carry a weight of 200N with the help of two ropes fixed to the weight one rope is inclined at 45^0 and the other at 30^0 with the vertices. Find the tension in each rope.

3. Find the moment of force at hinge, when the beam is applied with a force 100N at a distance of 500mm from the hinge.

4. Find the couple of force 15KN having an arm of couple 200MM

SHORT ANSWER QUESTIONS.

1. Define the following terms.
   a) Force b)Vector c) Scalar d)Resultant e) Equilibrant
2. State the characterstics of force.
3. State the Law of trinagle of forces.
4. State Lami's theorem
5. State law of polygon of forces.
6. State parallelogram Law of forces.
7. Define moment of force.
8. Define couple.

ESSAY TYPE QUESTIONS

1. State and explain classification of forces.
2. Find the resultant of two forces acting at a point by Law of paral -lelogram of forces.
3. Define couple, moment of couple, characteristics of couple.

3
CENTROID AND MOMENT OF INERTIA

3.1 Centroid

A body is made up of infinite number of minute particles. Every particle of a body is attracted by the earth towards its center. The force of attraction which is proportional to the mass of the particle acts vertically downwards and is known as weight of the body. The weights of individual particles act downward and form a system of like parallel forces. The resultant of this system of forces known as the weight of the body and acts through a fixed point in the body in position the body is held. The fixed point is called as centre of gravity of the body. The plane figures like triangle, rectangle, circle etc. have only areas, but no mass. The center of area of such figures is known as centroid of the body.

3.2 Method of locating centroid of simple figures.

The centroid of a surface may be found out by one of the following methods:

a) Geometrical consideration
b) Method of moments
c) Graphical method

a) Geometrical consideration : The centre of gravity of simple figures may be found out from the geometry of the figure.

1) The c.g. of uniform rod is at its middle point.
2) The c.g of rectangle (or of a parallelogrm) is at the point, where the diagonals meet each other.

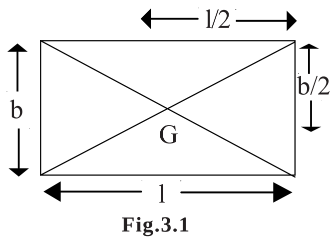


Fig.3.1

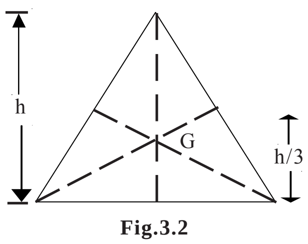


Fig.3.2

3) The c.g. of a triangle is at the point, where the three medians (a median is a line connecting the vertex and the middle point of the opposite side) of the triangle meet.

4) The c.g. of a Semi-circle is at a distance of $4r/3\pi$ from its base measured along the vertical radius. The c.g. of a circle is at its geo metric centre.

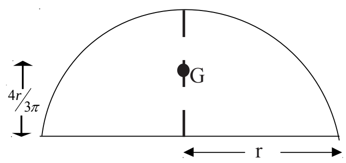


Fig.3.3

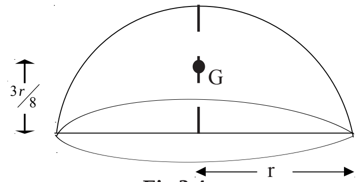


Fig.3.4

5) The c.g. of a hemi sphere is at a distance of $3r/8$ from its base measured along the vertical radius.

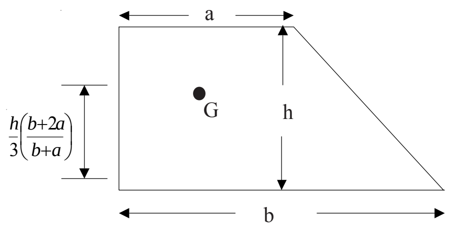


Fig.3.5

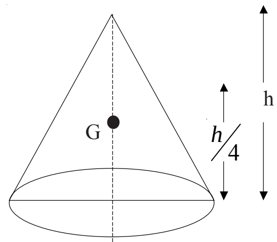


Fig.3.6

6) The c.g. of a trapezium with parallel sides a and b is at a distance of

$h/3\left(\frac{b+2a}{b+a}\right)$ measured from the side b.

7) The c.g. of a solid cone is at a distance of h/4 from its base, measured along the vertical axis.

8) The c.g. of a parabola is at 3h / 8 from axis and 3L/5 from vertex.

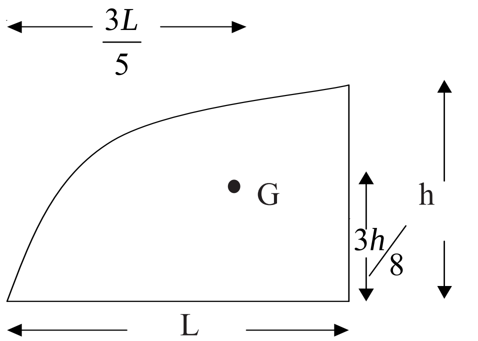


Fig.3.7

b) Method of moments:

Consider a body of area A and of uniform thickness. Divide the area into small areas A_1, A_2, A_3.... etc., Let the c.g. of small areas are (X_1, Y_1), (X_2, Y_2), (X_3,Y_3),.......... etc., from the fixed point O. Equating the moment of total area to the moments of small areas about OY

$$A\bar{x} = A_1 X_1 + A_2 X_2 + ... = \sum AX$$

$$\therefore \quad \bar{X} = \frac{\sum AX}{A}$$

$$Similarly\, \bar{Y} = \frac{\sum AY}{A}$$

Where $A = A_1 + A_2 + A_3 + ...$

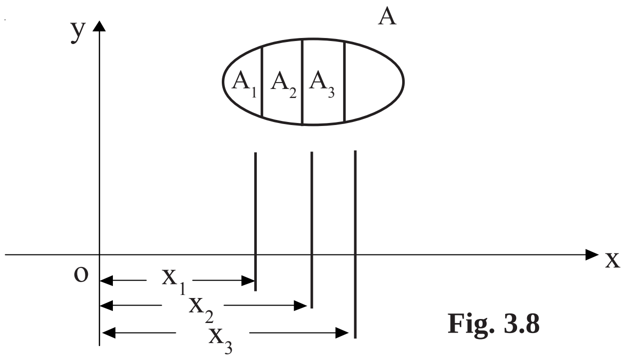


Fig. 3.8

c) Graphical method

This will be discussed in Graphic Statics chapter.

3.3 Axis of reference

The c.g. of a body is always calculated with reference to some assumed axis known as axis of reference. The axis of reference of plane figures, is generally taken as the lowest line of the figure for calculating $\bar{Y}$, and the left line of the figure for calculating $\bar{X}$.

3.4 Axis of symmetry

Sometimes the body may be symmetrical about X-X axis or Y-Y axis. In such cases we have to calculate either $\bar{X}$ or $\bar{Y}$ only. This is due to the reason that the c.g. of the body will lie on the axis of symmetry.

Example 3.1 Find the centre of gravity of a T-Section 12X20X 6cm..

Solution:

The section is symmetrical about y-axis. The c.g. of the section will lie on this axis. Divide the section into two parts. Take bottom line as axis of reference.

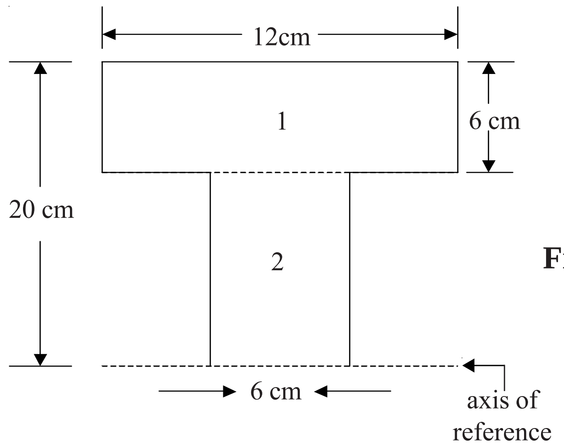


Fig.3.9

a_1 = area of part 1 = 12 x 6 = 72 Cm^2

y_1 = c.g. of part 1 w.r.t. bottom line = $\left(20-\frac{6}{2}\right)$ = 17cm

a_2 = area of part2 = (20-6) x6 = 84 cm^2

$$y_2 = \frac{20-6}{2} = 7\,cm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{72 \times 17 + 84 \times 7}{72 + 84}$$

$$= \frac{1224 + 588}{156} = 11.61\,cm$$

Example 3.2 Find the centroid of an I-section:

Top Flange 80x 20 mm

Web 120x20mm

Bottom Flange 120 x 20mm

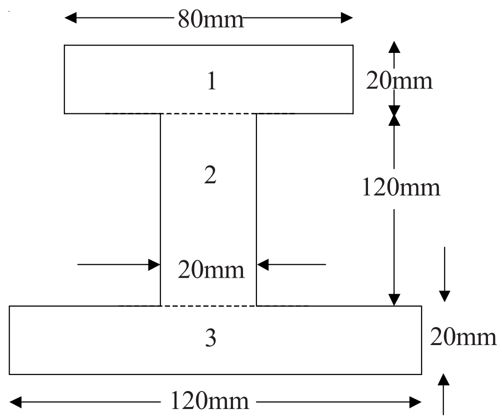


Fig.3.10

Solution:

The section is symmetrical about y - axis. The c.g. of the section will lie on this axis. Divide the section into 3 parts. Take bottom line as axis of reference to find $\overline{Y}$.

a_1 = area of part 1 = 80 x 20 = 1600 mm^2

$y_1 = \frac{20}{2} + 120 + 20 = 150$ mm

a_2 = area of part 2 = 120 x 20 = 2400 mm^2

$y_2 = \frac{120}{2} + 20 = 80$mm

a_3 = area of part 3 = 120 x 20 = 2400 mm^2

$Y_3 = \frac{20}{2} = 10$mm

$$\therefore \overline{y} = \frac{a_1 y_1 + a_1 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1600 \times 150 + 2400 \times 80 + 2400 \times 10}{1600 + 2400 + 2400}$$

$$= \frac{24000 + 192000 + 24000}{6400}$$

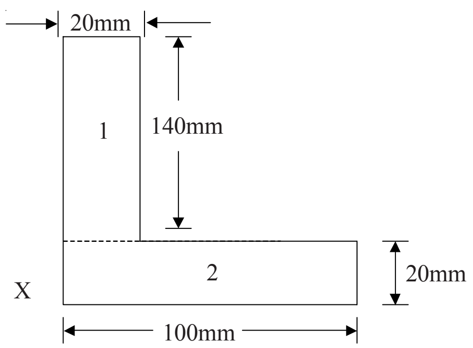
= 71.25 mm from bottom line.

Example 3.3 Find the centroid of an angular section shown below (I.P.E.March 1996)

Solution:

Here we have to find both $\overline{x}$ and $\overline{y}$ because the section is not symmetrical about x or y - axis. To find $\overline{y}$, take bottom line as axis of reference. To find $\overline{x}$, take left line as axis of reference. Divide the figure into two parts.

Fig.3.11



a_1 = area of part 1 = 140 x 20 = 2800 mm^2

a_2 = area of part 2 = 100 x 20 = 2000 mm^2

y_1 = centroid of part 1 from X-X $= \frac{140}{2} + 20 = 90mm$

y_2 = centroid of part 2 from X-X = $\frac{20}{2} = 10mm$

x_1 = centroid of part 1 from y-y = $\frac{20}{2} = 10mm$

x_2 = centroid of part 2 from y-y = $\frac{100}{2} = 50mm$

Now,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{2800 \times 90 + 2000 \times 10}{2800 + 2000}$$

$$= \frac{252000 + 20000}{4800} = 56.66mm$$

$$\bar{x} = \frac{a_1 x_1 + a_2\, x_2}{a_1 + a_2} = \frac{28000 \times 10 + 2000 \times 50}{2800 + 2000}$$

$$= \frac{28000 + 10000}{4800} = 26.66mm$$

Example 3.4 A masonry dam is trapezoidal in section with one face vertical. Top width is 5m and bottom width is 10m and height is 18m. Find the distance of centroid from vertical face and from the bottom line.

Solution:

To find $\bar{y}$, take bottom line (O-X) as axis of reference. To find $\bar{x}$, take vertical face (O-Y) as axis of reference. Divide the figure into two parts.

a_1 = area of part 1 = 18 x 5 = 90m^2
a_2 = area of part 2 = 1/2 x 5 x 18 = 45 m^2

y_1 = c.g. of part 1 = $\frac{18}{2} = 9m$

y_2= c.g. of part 2 from O-X = $\frac{18}{3}$ = 6m

x_1= c.g. of part 1 from O-Y = $\frac{5}{2} = 2.5m$

x_2= c.g. of part 2 from O-Y = $5 + \frac{5}{3} = 6.67m$

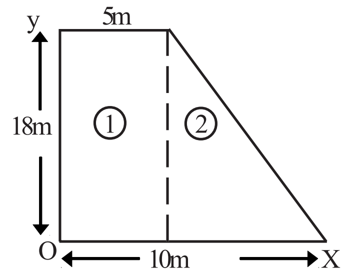


Fig.3.12

$$\therefore \text{Distance of Centroid from O} - \text{X} = \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\bar{y} = \frac{90 \times 9 + 45 \times 6}{90 + 45} = 8\text{m}$$

$$\therefore \text{Distance of Centroid from O} - \text{Y} = \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$\bar{x} = \frac{90 \times 2.5 + 45 \times 6.67}{90 + 45} = 3.89\text{m}$$

Example 3.5

Determine the position of centroid of a lamina shown below.

Solution:

The section is symmetrical about y - axis. So centroid lies on this axis. Find $\bar{y}$. To find $\bar{y}$ take base line as axis of reference. Divide the section into two parts.

$$a_1 = \frac{1}{2} \times 100 \times 60 = 3000\text{mm}^2$$

$$a_2 = 100 \times 100 = 10000\text{mm}^2$$

$$y_1 = 100 + \frac{60}{3} = 120\text{mm}$$

$$y_2 = \frac{100}{2} = 50\text{mm}$$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 120 + 10000 \times 50}{3000 + 10000}$$

$= 66.15$ mm above the base line.

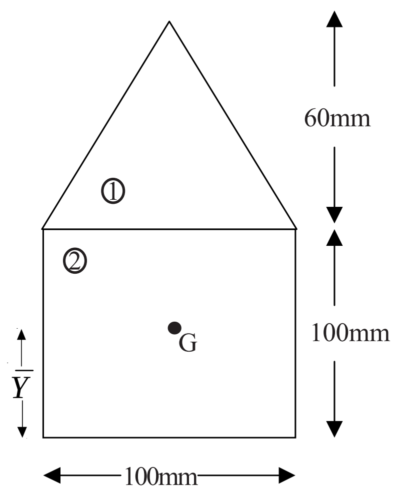


Fig.3.13

Example 3.6 Determine the centroid of the channel section 200x100x10mm

Solution:

The section is symmetrical about X-X axis. So find $\bar{x}$. To find $\bar{x}$, take left vertical line (o-y) as axis of reference. Divide the section into 3 parts.

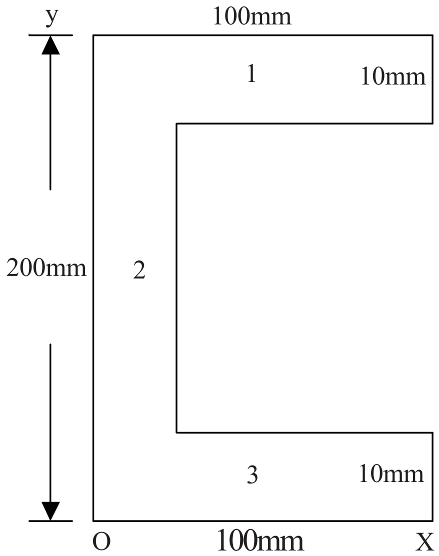


Fig.3.14

$a_1 = 100 \times 10 = 1000mm^2$

$x_1 = \frac{100}{2} = 50mm$ from $O - Y$

$a_2 = 180 \times 10 = 1800mm^2$

$x_2 = \frac{10}{2} = 5$ mm from $O - Y$

$a_3 = 100 \times 10 = 1000mm^2$

$x_3 = \frac{100}{2} = 50mm$ from $O - Y$

$$\therefore \quad \bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{1000 \times 50 + 1800 \times 5 + 1000 \times 50}{1000 + 1800 + 1000}$$

$$= 28.68mm \ from O - Y$$

$$\therefore \bar{y} = \frac{200}{2} = 100mm \ from O - X$$

Example 3.7

Find the centroid of the square plate with a semi circle cutout shown below.

Solution

The section is symmetrical about y- axis. Take line DC as axis of reference. Divide the plate into two parts.

a_1 = area of rectangle

= $200 \times 200 = 40000 \ mm^2$

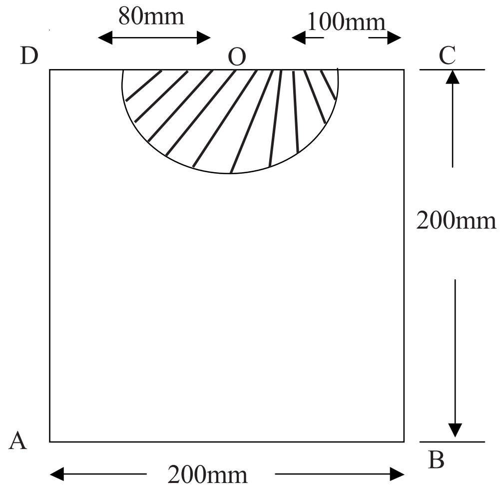


Fig.3.15

$$a_2 = area\ of\ semi-circle = \frac{1}{2}\left(\frac{\Pi}{4}\times 160^2\right) = 10048\ mm^2$$

$$y_1 = c.g.of\ \ part\,1\ from\ line\ DC = \frac{200}{2} = 100mm$$

$$y_2 = c.g.of\ \ part\,2\ \ from\ line\ DC = \frac{4r}{3\Pi}$$

$$= \frac{4\times 80}{3\times 3.14} = 33.97mm$$

$$\therefore \quad \bar{y} = \frac{a_1 y_1 - a_2\, y_2}{a_1 - a_2}$$

$$= \frac{40000\times 100 - 10048\times 33.97}{40000 - 10048}$$

$$= 122.15mm\ \ from\ top\ line\ DC$$

Example 3.8 Find the centroid of the circular section shown below.
Solution:

$$\text{Area of total section } (a_1) = \frac{\Pi\times 120^2}{4}$$

$$= 11304\text{mm}^2$$

$$\text{Area of Circular hole } (a_2) = \frac{\Pi\times 40^2}{4}$$

$$= 1256\text{mm}^2$$

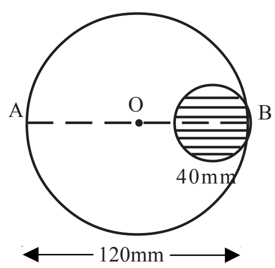


Fig.3.16

The section is symmetrical about x- axis.

$$x_1 = \text{c.g. of total section from A} = \frac{120}{2} = 60\text{mm}$$

$$x_2 = \text{c.g. of Circular hole from A}$$

$$= \frac{40}{2} + (120 - 40) = 100\text{mm from A}$$

$$\therefore \bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{11304 \times 60 - 1256 \times 100}{11304 - 1256}$$

$$= \frac{552641}{10048} = 55 \text{ mm from A.}$$

Exercise 3.1

1. Find the centroid of T-Section with top flange 100x20 mm and web 100x20mm (Ans: 8mm from botton)

2. Find the centroid of the sections shown below.

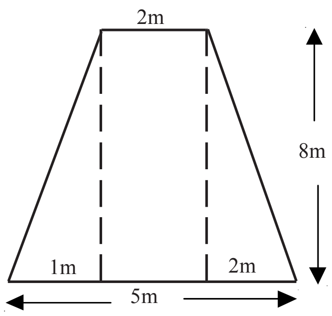


Fig.A

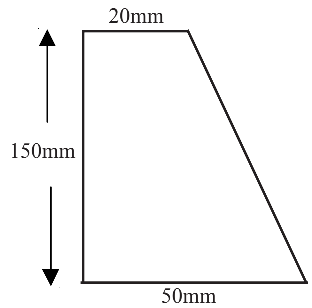


Fig.B

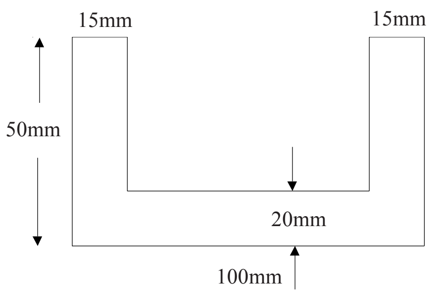


Fig.C

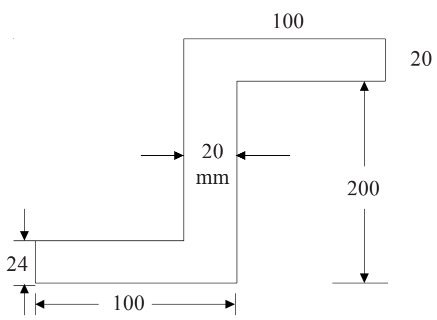


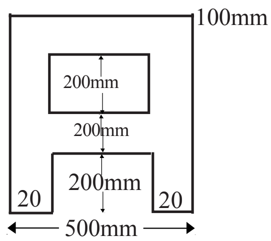
Fig.D

Ans: a) $\bar{y}$ = 3.429 m from bottom edge.
$\bar{x}$ = 2.286m from left edge.
b) $\bar{y}$ = 61.11 mm from bottom edge
$\bar{x}$ = 18.57mm from left edge
c) $\bar{y}$ = 17.75 mm from the bottom edge
$\bar{x}$ = 50mm from left edge
d) $\bar{y}$ = 106.44mm from bottom edge
$\bar{x}$ = 87.97 mm from left Edge.

3. Find the centroid of an I - section whose
Top Flange : 150 x 50 mm
Web : 400 x 50 mm
Bottom flange : 300 x 100 mm
(Ans: $\bar{y}$ = 198 .90 mm from base)

4. Find the centroid of an I - Section whose
Top flange : 150 x 20 mm
web : 150 x 20 mm
Bottom flange : 200 x 20 mm
(Ans: $\bar{y}$ =86.5mm from base)

5. Find the centroid of the section shown below.
(Ans: $\bar{y}$ =332.77mm From bottom edge)



Short Answer Questions

1. Define center of gravity.
2. State the position of centroid for the following sections.
a) Rectangle b) Triangle c) circle d) Semi - Circle
3. Find the centroid of a triangle of base 80 mm and height 120 mm from the base. (Ans. 40mm)
4. State the methods of locating centroid.
5. Find the position of centroid of the section of one side vertical trapezoidal section of top width 2m, base 8m and height 9m.
(Ans. 2.80m from vertical face)

3.5 Moment of Inertia

Newton's first law of motion states that everybody is in its state of rest or of uniform motion along a straight line unless an external force acts on it to change the state. This property of a body to maintain its state is called "Inertia" It is changed by an external force acting on the body and is proportional to the mass of the body.

Consider a lamina of area A. The lamina may be divided into number of small components of areas a_1, a_2, a_3, etc. be at distances y_1,y_2,y_3, etc. from the axis OX and at distances x_1,$x_2$$x_3$, etc., from the axis OY.

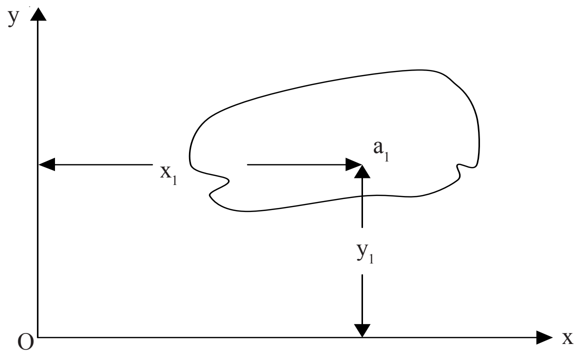


Fig 3.17

The product of area and perpendicular distance between the point is called as first moment of area about the point. If this is again multiplied by the distance, then it is called second moment of area (or) moment of Intertia (M.I). Its unit in SI System is mm^4.

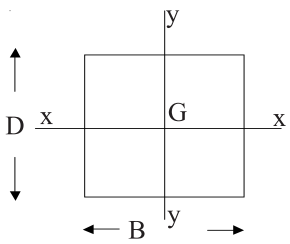
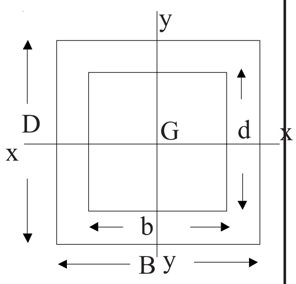
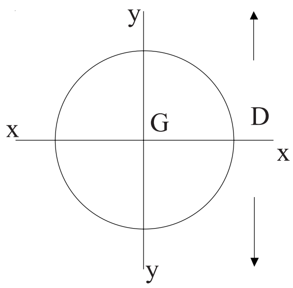
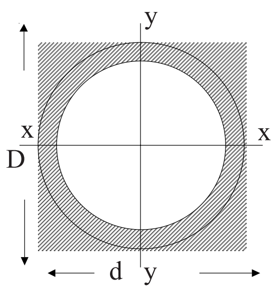
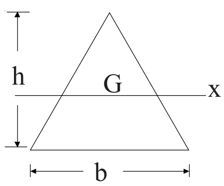
$$M.I.about\ x-axis \quad = a_1 y_1^2 + a_2 y_2^2 + a_3\ y_3^2 + ...etc.$$

$$= \sum_{i=1}^{i=n} a_i\ y_i^2$$

$$M.I.about\ y-axis \quad = a_1 x_1^2 + a_2 x_2^2 + a_3\ x_3^2 + ...etc.$$

$$= \sum_{i=1}^{i=n} a_i\ x_i^2$$

3.6 Moment of Inertia of plane figures:

Name	Shape	M.I. About X-axis(I_{xx})	M.I.about y-axis(I_{yy})
Rectangle (Solid)		$\frac{BD^3}{12}$	$\frac{DB^3}{12}$
Rectangle (Hollow)		$\frac{BD^3}{12} - \frac{bd^3}{12}$	$\frac{DB^3}{12} - \frac{db^3}{12}$
Circle (Solid)		$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$
Circle (Hollow)		$\frac{\pi}{64}\left(D^4 - d^4\right)$	$\frac{\pi}{64}\left(D^4 - d^4\right)$
Triangle		$\frac{bh^3}{36}$ (about centroidal axis)	$\frac{bh^3}{12}$ (about the base)

3.7 Radius of Gyration

The radius of gyration of a given area about a given axis is defined as the distance from the given axis.

$$I = AK^2$$

where I = moment of Inertia

A = Area of the lamina

K = Radius of Gyration

$$\therefore \quad K = \sqrt{\frac{I}{A}}$$

Units for K in S.I.System is mm

3.8 Parallel Axis Theorem

The moment of inertia of a plane area about any axis parallel to centroidal axis and at a distance of 'h' from the centre of gravity is the sum of moment of inertia about centroidal axis and the product of the area and square of the distance betwen the two axes.

$$\boxed{I_{AB} = I_{GG} + Ah^2}$$

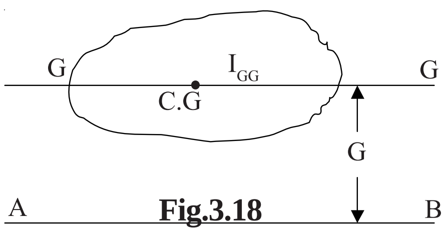


Fig.3.18

Where I_{AB} = M.I. of the area about an axis AB

I_{GG} = M.I. of the area about its C.G.

A = Area of the lamina

h = distance between C.G. and axis AB.

3.9 Perpendicular Axis Theorem

Let I_{0x} and I_{oy} be the moments of Inertia of a plane about two mutually perpendicular axes OX and OY as shown in figure. The moment of inertia about an axis OZ, perpendicular to OX and OY is given by

$$\boxed{I_{OZ} = I_{OX} + I_{OY}}$$

For circular section,

$I_{OX} = I_{OY}$

$\therefore \quad I_{OZ} = 2\, I_{OX}$

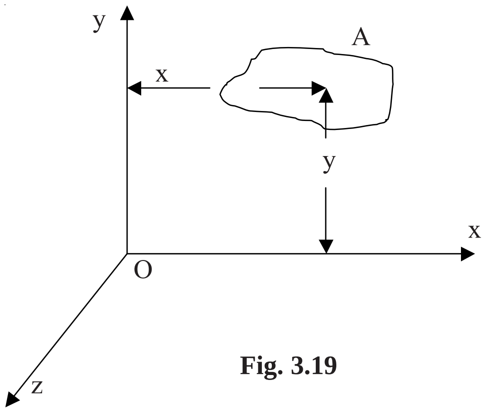


Fig. 3.19

3.10 Polar Moment of Inertia

The moment of inertia of an area, with respect to an axis perpendicular to its plane is called polar moment of Inertia. This property is very useful in design of shafts. It is usally denoted by J.

M.I. of a circular section = $I_{0x} = I_{oy} = \dfrac{\Pi d^4}{64}$

By perpendicular axes theorem $I_{OZ} = I_{OX} + I_{OY} = 2 \times \Pi d^4/64 = \dfrac{\pi d^4}{32}$

and $K_{Oz} = \sqrt{I_{oz}/A} = \sqrt{\dfrac{\Pi d^4/32}{\Pi d^2/4}} = \dfrac{d}{\sqrt{8}}$

Example 3.9 Find the moment of Inertia of a rectangle 200 mm X 600mm about its base.

Solution:

Breadth B= 200 mm
Depth D = 600 mm

h = $\dfrac{600}{2} = 300mm$

M.I. about centre of gravity

$I_{CG} = \dfrac{BD^3}{12} = \dfrac{200 \times 600^3}{12}$

$= 36 \times 10^8 \text{ mm}^4$

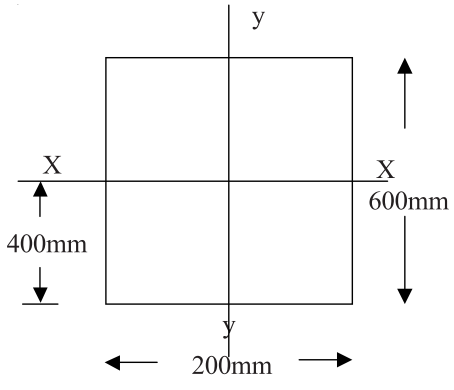


Fig. 3.20

M.I. about its base (I_{base}) = I_{CG} + A h^2

= $36 \times 10^8 + (200 \times 600)(300)^2$

= $36 \times 10^8 + 108 \times 10^8 = 144 \times 10^8$ mm^4

Example 3.10 Find the moment of inertia of a triangular section base width 100mm and height 60mm about its base.

Solution:

Base of the traingle (b) = 100mm

Height of the triangle(h) = 60mm

M.I. about C.G. (I_{CG}) = $\frac{bh^3}{36}$

= $\frac{100\times60^3}{36} = 6\times10^5\ mm^4$

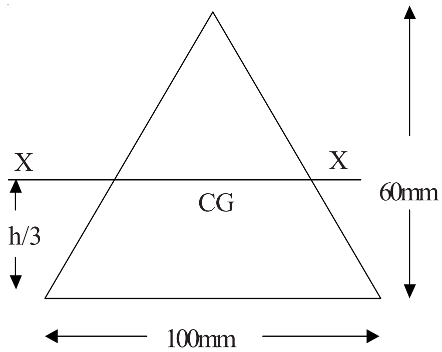


Fig 3.21

M.I. about its base (I_{base}) = I_{CG} + Ay^2

Where A = Area of triangle

y = Distance of base from centroid

$$\therefore I_{base} = 6\times10^5 + \left(\frac{1}{2}\times100\times60\right)\left(\frac{60}{3}\right)^2$$

$$= 6\times10^5 + 12\times10^5 = 18\times10^5\ mm^4$$

Example 3.11 Find M.I., Polar M.I. and radius of gyration of a hollow circular section of 100mm external dia and 10mm thick.

$\overline{Y}$

Solution :

External Diameter = 100mm

Internal Diameter = 100 - 2 x thickness

= 100-2x10 = 80 mm

M.I. of hollow circular section

I_{ox} (or) I_{oy} =

$$\frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi}{64}\left(100^4 - 80^4\right)$$

$$= 2.89\times10^6\ mm^4$$

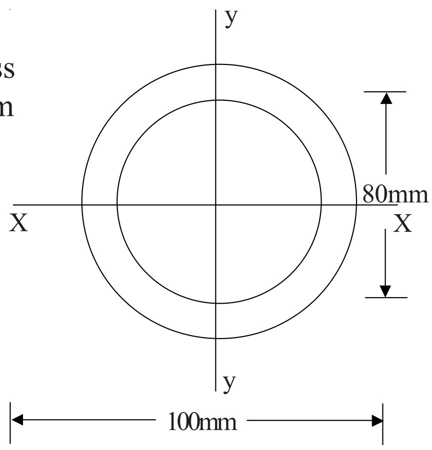


Fig. 3.22

Polar M.I. (I_{oz})= I_{ox} + I_{oy}

= $2.89 \times 10^6 + 2.89 \times 10^6$

= 5.78×10^6 mm^4

Area (A) $= \frac{\pi}{64}\left(D^2 - d^2\right)$

$= \frac{\pi}{64}\left(100^2 - 80^2\right) = 176.79 \text{ mm}^2$

radius of gyration= $\sqrt{\frac{I_{ox}}{A}}$

$= \sqrt{\frac{2.89 \times 10^6}{176.79}} = 127.86 \ mm$

Example 3.12 Find the M.I. of a T - section about its centroidal x - axis shown in fig. 3.23.

Solution :

(a_1) area of part 1 = 300 x 100
= 30000 mm^2

$y_1 = 200 + \frac{100}{2} = 250 mm \ from\ bottom$

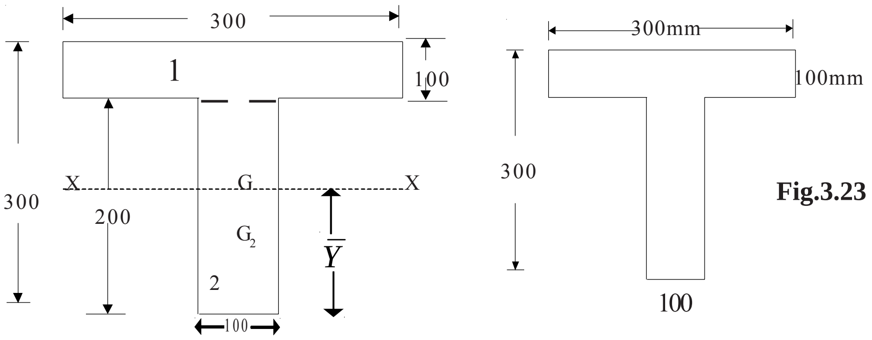


Fig.3.23

(a_2) Area of part 2 = 200 x 100 = 20000 mm^2

$y_2 = \frac{200}{2}$ = 100 mm from bottom

$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$= \frac{30000 \times 250 + 20000 \times 100}{30000 + 20000} = 190\text{mm from bottom}$

Distance of centriod of part 1 from XX axis

$h_1 = \left(200 + \frac{100}{2}\right) - 190 = 60 mm$

Distance of centroid of part 2 from XX-axis

$$h_2 = 190 - \frac{200}{2} = 90\,mm$$

M.I. of part 1 from X - X axis

$$I_{x-x_1} = I_{CG1} + a_1 h_1^2$$

$$= \frac{300 \times 100^3}{12} + 30000 \times 60^2 = 133 \times 10^6 \ mm^4$$

M.I. of part 2 from x - x axis

$$I_{x-x_2} = I_{CG2} + a_2 h_2^2$$

$$= \frac{100 \times 200^3}{12} + 20000 \times 90^2 = 228.7 \times 10^6 \ mm^4$$

Now M.I of the whole section about its centriodal x-x axis

$I_{xx} = I_{x-x_1} + I_{x-x_2} = 133 \times 10^6 + 228.7 \times 10^6$

$= 361.70 \times 10^6 \ mm^4$

Example 3.13

Find the moment of inertia of the L - Section shown in Fig. 3.24 about its horzonal and verticl centroidal axis. (I.P.E. march 1997)

Solution:

Divide the figure into two parts.

Area of part 1 (a_1) = 140 x 20 = 2800 mm^2

$$y_1 = \frac{140}{2} + 20 = 90mm \ from\, OX$$

$$x_1 = \frac{20}{2} = 10mm \ from\, oy$$

Area of part 2 (a_2) = 100 x 20 = 2000 mm^2

$$y_2 = \frac{20}{2} = 10\,mm\ from\,OX$$

$$x_2 = \frac{100}{2} = 50\,mm\ from\ OY$$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{2800 \times 90 + 2000 \times 10}{2800 + 2000} = 56.67mm$$

$$\therefore \bar{x} = \frac{a_1\,x_1 + a_2\,x_2}{a_1 + a_2} = \frac{2800 \times 10 + 2000 \times 50}{2800 + 2000} = 26.67mm$$

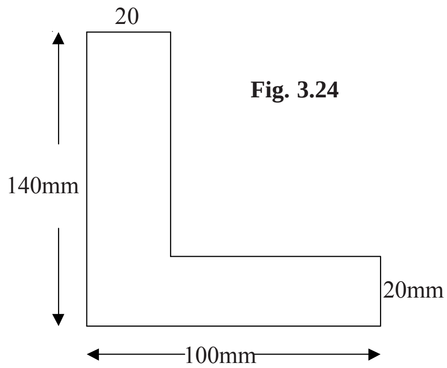
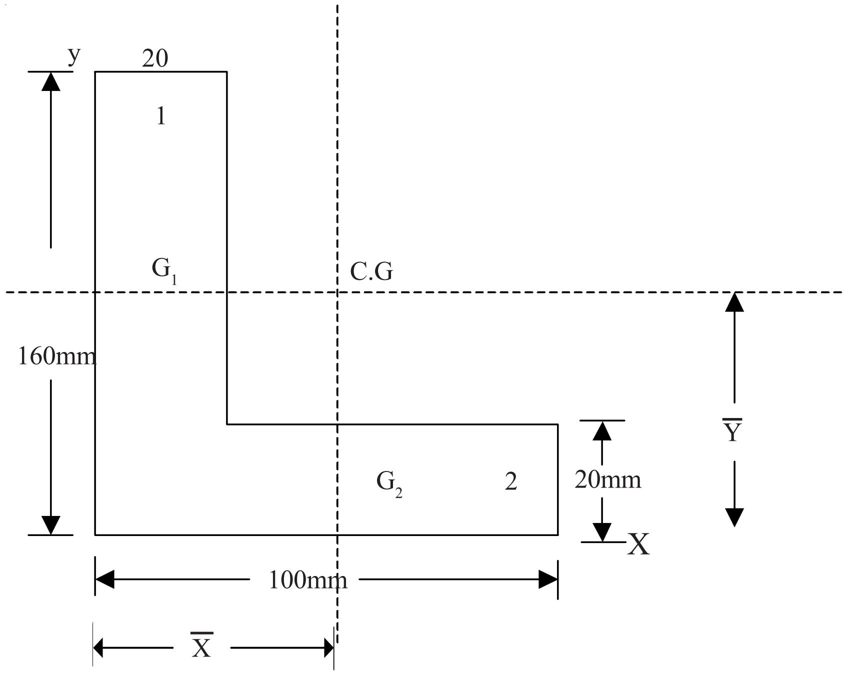


Fig. 3.24



To find $I_{X\text{-}x}$

Distance of centroid of part 1 from x - axis

$$h_1 = \left(\frac{140}{2} + 20\right) - 56.67 = 33.33mm$$

Distance of controid of part 2 from x - axis

$$h_2 = 56.67 - \frac{20}{2} = 46.67mm$$

$$I_{X-X_1} = I_{CG_1} + a_1 h_1^{\,2}$$

$$= \frac{20 \times 140^3}{12} + (140 \times 20)(33.33)^2 = 7.68 \times 10^6\,mm^4$$

$$I_{X-X_2} = I_{CG_2} + a_2 h_2^{\,2}$$

$$= \frac{100 \times 20^3}{12} + (100 \times 20)(46.67)^2 = 4.42 \times 10^6\,mm^4$$

$\therefore$ M.I. about x – axis

$$I_{XX} = I_{X-X_1} + I_{X-X_2}$$

$$= 7.68 \times 10^6 + 4.42 \times 10^6 = 12.10 \times 10^6\,mm^4$$

To find I_{y-y}

Distance of centroid of part 1 from y – axis

$$h_1 = 26.67 - \frac{20}{2} = 16.67mm$$

Distance of centroid of part 2 from y - axis

$$h_2 = \frac{100}{2} - 26.67 = 23.33mm$$

$$I_{y-y_1} = I_{CG1} + a_1 h_1^{\,2}$$

$$= \frac{140 \times 20^3}{12} + (140 \times 20)(16.67)^2 = 8.71 \times 10^5\,mm^4$$

$$I_{y-y_2} = I_{CG_2} + a_2 h_2^{\,2} = \frac{20 \times 100^3}{12} + (100 \times 20)(23.33)^2 = 2.75 \times 10^6\,mm^4$$

$\therefore$ M.I about y–axis $I_{y-y} = I_{y-y_1} + I_{y-y_2}$

$$= 8.71 \times 10^5 + 2.75 \times 10^6 = 3.62 \times 10^6\,mm^4$$

The calculations can be tabulated as shown below for I_{xx}

Part No	I_{CG} mm^4	Area mm^2	Centroid Distance from X-X h mm	h^2 mm^2	$I_{CG} + ah^2$ mm^4
1	4.57 x 10^6	2800	33.33	110.88	7.68x10^6
2.	6.67x 10^4	2000	46.67	2179.08	4.42x10^6
					Total = 12.10 x 10^6

For I_{yy}

Part No	I_{CG} mm^4	Area a mm^2	Centroidal distance from y-y(n)	h^2	$I_{CG} + ah^2$ (mm^4)
1.	1.12×10^6	2800	16.67	277.89	8.71×10^5
2.	1.67×10^6	2000	23.33	544.28	2.75×10^6
			Total = 3.62×10^6		

Example 3.14 Find the M.I. and radius of gyration for the I - Section shown in fig. 3.25 about the centroidal axes.

Solution : Consider the section as a rectangle of 200 mmx 300 mm from which two rectangles of 90mm x 260mm are to be removed.

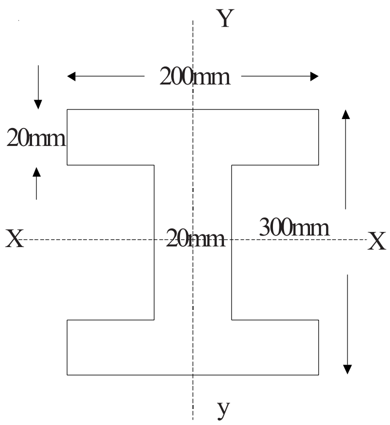
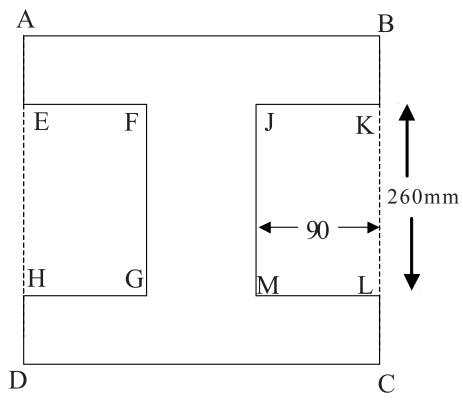


Fig 3.25



M.I. about XX axis

$$I_{XX} = \text{M.I. of rectangle ABCD} - \text{M.I. of rectangle EFGH \& JKLM}$$

$$= \frac{BD^3}{12} - 2\times\frac{bd^3}{12}$$

$$= \frac{200\times300^3}{12} - 2\times\frac{90\times260^3}{12}$$

$$= 18636 \times 10^4 \text{ mm}^4$$

M.I. about yy axis

I_{yy} = M.I. of rectangle ABKE & HLCD + M.I. of rectangle FJMG

$$= 2\left(\frac{20\times 200^3}{12}\right)+\frac{260\times 20^3}{12}$$

= $26.67 \times 10^6 + 17.33\ 10^4 = 26.84 \times 10^6\ mm^4$

Area of the section (A)=200 x 300 - 2 (90 x 260) = 13200 mm^2

= BxD - 2(bxd)

Radius of gyration about XX - axis

$$K_{xx}=\sqrt{\frac{I_{xx}}{A}}=\sqrt{\frac{18636\times 10^4}{13200}}=\sqrt{14118.18}=118.81mm$$

Radius of gyration about yy - axis

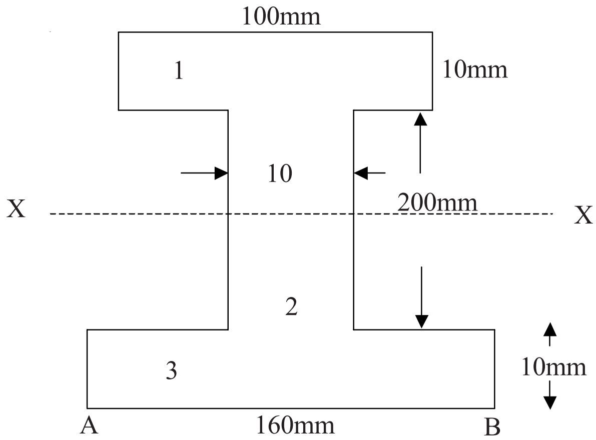
$$K_{yy}=\sqrt{\frac{I_{yy}}{A}}=\sqrt{\frac{26.84\times 10^6}{13200}}=\sqrt{2033.33}=45.09mm$$

Example 3.15 Find moment of inertia of an I - section

Whose Top flange = 100 mm x 10 mm

Web = 200 mm x 10 mm

Bottom Flange = 160 mm x 10 mm



Solution:

Centroid:

Fig.3.26

The Section is symmetrical about y - axis. So find $\bar{y}$. To find $\bar{y}$ take bottom line AB as axis of reference. Divide the section into 3 parts.

a_1 = 100 x 10 = 1000mm^2

$y_1 = \frac{10}{2} + 200 + 10 = 215\ mm$ bottom line AB

$a_2 = 200 \text{ x } 10 = 2000\ mm^2$

$y_2 = \frac{200}{2} + 10 = 100\ mm$ from AB

$a_3 = 160 \text{ x } 10 = 1600\ mm^2$

$y_3 = \frac{10}{2} = 5\ mm$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{1000 \times 215 + 2000 \times 110 + 1600 \times 5}{1000 + 2000 + 1600}$$

$= 96.30$ mm

M.I. about xx

$I_{x\text{-}x1}$ = M.I. of part 1 about x - x

$= I_{CG1} + A_1 h_1^2$

$$= \frac{100 \times 10^3}{12} + (100 \text{ x } 10)(215\text{-}96.20)^2 = 14.09 \text{ x } 10^6\ mm^4$$

$I_{x\text{-}x2}$ = M.I. of part 2 about x - x = $I_{CG2} + A_2\ h_2^2$

$$\frac{10 \times 200^3}{12} + (200 \times 10)(110 - 96.30)^2 = 7.04 \times 10^6\,\text{mm}^4$$

I_{X-X3} = M.I. of part 3 about X − X = $I_{CG3} + A_3\ b_3^2$

$$= \frac{160 \times 10^3}{12} + (160 \times 10)(96.30 - 5)^5 = 13.35 \times 10^6$$

$$\therefore I_{X-X} = I_{X-X1} + I_{X-X2} + I_{X-X3} = 14.09 \times 10^6 + 7.04 \times 10^6 + 13.35 \times 10^6$$

$$= 34.48 \times 10^6\ \text{mm}^4$$

M.I. about Y - Y

y - axis passing through centroid, $\therefore h_1 = h_2 = h_3 = 0$

I_{y-y1} = M.I. of part 1 about y − y

$= I_{CG1} + A_1\ h_1^2$

$$= \frac{10 \times 100^3}{12} = 0.83 \times A_1\ h_1^2$$

I_{y-y2} = M.I. of part 2 about y-y = $\frac{200\times10^3}{12} = 0.017\times10^6\,mm^4$

I_{y-y3} = M.I. of part 3 about y - y

= $I_{CG3} = \frac{10\times160^3}{12} = 3.41\times10^6$

$\therefore I_{y-y}$ = $I_{y-y1} + I_{y-y2} + I_{y-y3} = 4.257 \times 10^6\ mm^4$

Example 3.16 Find the M.I. of a rectangular plate of size 200 x 300 mm with a hole of 120 mm diameter as shown in fig.3.27 about AA and BB axis.

Solution :

The section is summetrical about B-B axis and the centroid lies on this axis at a distance of $\frac{300}{2}$ = 150mm

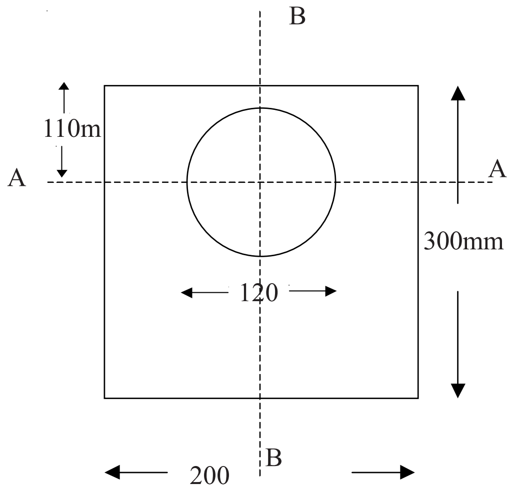


Fig.3.27

Distance between C.G. and A-A (h) = 150 - 110 = 40mm

M.I. of rectangle about A-A

$I_{Rectangle}$ = $I_{CG} + Ah^2$

= $\frac{200\times300^3}{12} + (200\times300)40^2$

= $4.5 \times 10^8 + 0.96 \times 10^8 = 5.46 \times 10^8\ mm^4$

M.I. of circular hole about AA

$$I_{circle} = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 120^4 = 0.10 \times 10^8 \ mm^4$$

M.I. of rectangular plate with circular hole about AA

$$I_{AA} = I_{Rectangle} - I_{Circle}$$

$$= 5.46 \times 10^8 - 0.1 \times 10^8 = 5.36 \times 10^8 \ mm^4$$

M.I. of rectangular plate with circular hole about BB

$$I_{BB} = \frac{300 \times 200^3}{12} - \frac{\pi (120)^4}{64}$$

$$= 2 \times 10^8 - 0.10 \times 10^8 = 1.9 \times 10^8 \ mm^4$$

Example 3.17

Find the M.I. of a built - up section made of standard ISLB 100 and and one plate 80 x 10 mm is shown in fig. 3.28

Solution:

The properties of the I - section are given in structural table as area of cross-section (A) = 1021mm^2 M.I. along X-X = I_{X-X} = 168 X 10^4 mm^4
Centroid : The section is symmetrical about y - y. Divide the section into two parts.

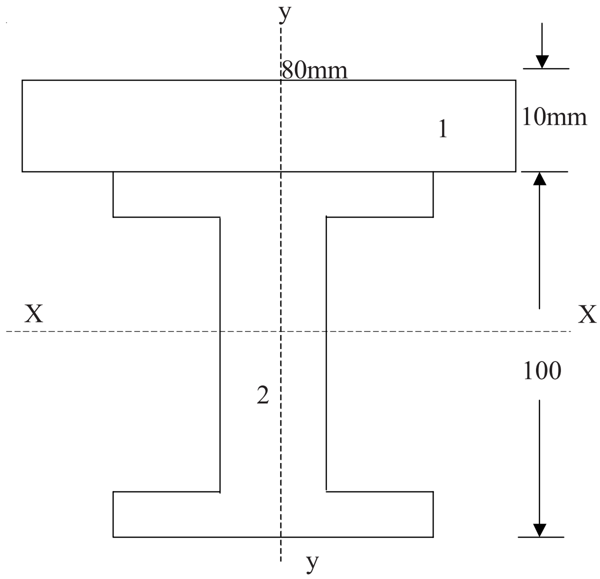


Fig.3.28

$\therefore \quad \bar{y} = \dfrac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \dfrac{(80\times10)\left(100+\dfrac{10}{2}\right)+1021\times\dfrac{100}{2}}{800+1021}$

$= 74.16\,mm \;\; from\,bottom\ line$

M.I. along X-X

$I_{X\text{-}X1}$ = M.I. of top rectangular plate

= $I_{CG1} + A_1 h_1^2$

= $\dfrac{80\times10^3}{12}+(80\times10)(105-74.16)^2 \;(\therefore h_1 = y_1 - \bar{y})$

= $7.67 \times 10^5\ mm^4$

$I_{x\text{-}x2}$ = M.I. of I- section

= $I_{CG2} + A_2 h_2^2$

= $168 \times 10^4 + 1021\ (74.16 - 50)^2 \;(\therefore h_2 = \bar{y} - y_2)$

= $22.7 \times 10^5\ mm^4$

$\therefore \quad I_{x-x} = I_{x-x1} + I_{x-x2}$

= $7.67 \times 10^5 + 22.7 \times 10^5 = 30.4 \times 10^5 mm^4$

Exercise 3.2

1. Find M.I. along X-X and y-y for the section shown in fig.3.29

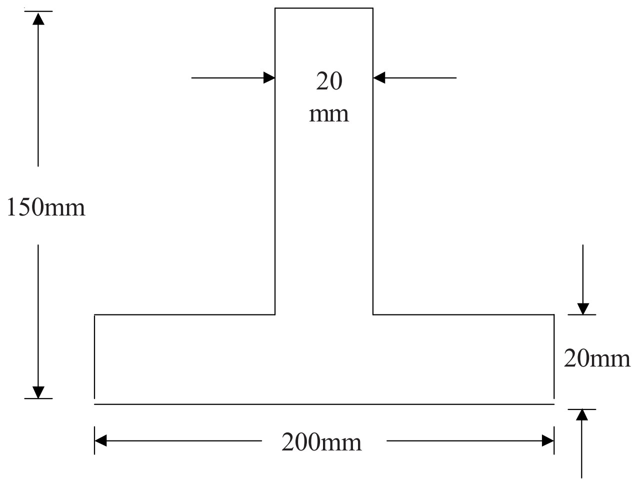


Fig.3.29

2. Find the M.I. of a T- section having flange 150 x 20mm and web 150 x 20mm about centroidal X-X and Y-Y

3. Find the M.I. and least radius of gyration of I - section whose

Tope flange	:	500 x 25mm
Web	:	400 x 25 mm
Bottom Flange	:	200 x 25 mm

(Ans : I_{xx} = 8.32 x 10^8 mm^4, I_{yy} = 2.78 x 10^8 mm^4
r_{min} = 100.46 mm)

4. Find M.I. along centroidal X-X and y-y for the built - up section with R.S.J (I- section) 200mm x 140 mm and with top plate 200x 10mm The properties of R.S.J. are

Area of cross-section = 3670 mm^2
I_{XX} = 26.24 x 10^6 mm^4
I_{yy} = 3.29 x 10^6 mm^4

(Ans. I_{xx} = 70.38 x 10^6 mm^4 I_{yy} = 16.62 x 10^6 mm^4)

5. Find the M.I. and radius of gyration of a section shown in fig. 3.30 about its base. (Ans. I_{base} = 4x 10^6 mm^4)

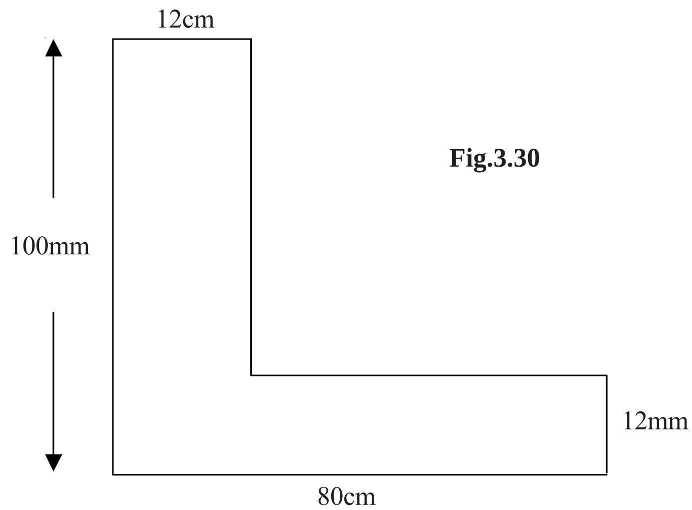


Fig.3.30

6. A built up section is made up of an ISHB 400 and two flat plates of 300x 20mm one at top and another at bottom. Find M.I. about its centroidal X-X and y-y axes.

The properties of I-section are
area = 10466 mm^2

(Ans. I_{xx} = 2.88 x 10^8 mm^4 I_{yy} = 0.28 x 10^8 mm^4)

Short Answer questions

1. Define moment of Inertia.
2. Define radius of gyration.
3. Define polar moment of Inertia.
4. Write the values of M.I. for the following Sections : (a) Rectangle (b) Triangle (c) circle. (d) hollow circle section
5. State parallel axis theorem
6. State perpendicular axis theorem

4
SIMPLE STRESSES AND STRAINS

4.1 General

It is necessary to study the properties of the materials and behavior of the materials under loads for design of structure in economical and efficient way. In general materials are classified into elastic, plastic and rigid materials. An elastic material when applied with loads undergoes deformation in such a way that, the deformation disappears on the removal of the load. A plastic material when applied with loads undergoes deformation and the material will not return to its original shape and size when the load is removed. A rigid material when applied with loads does not undergo any deformation.

Elasticity:

A body is applied with loads undergoes deformation and the body produces some resistance to the deformation, when the load is removed, the resistance to deformation disappears and the body return back to its original dimensions. This is possible only, when the deformation is within a certain limit. This limit is called Elastic limit. The properties of the material of returning back to its original position after removal of the loads is called as elasticity.

4.2 Stress

When a body is applied with loads, it under goes deformation. But the body will produce internal resistance to deformation. This resistance per unit area to deformation is known as 'Stress'. Stress may be represented as force per unit area.

Mathematically $f = \frac{P}{A}$

Where	f	=	Stress
	p	=	Load (or) force acting on the body
	A	=	Cross sectional area of the body.

4.3 Strain

When a body is applied with load the length of the body change from ℓ to $\ell + \delta\ell$. The ratio of the change in length to the original length of the body is called strain.

Mathematically $e = \frac{\delta \ell}{l}$

Where e = strain

$\delta \ell$ = change in length of the body

l = original length of the body

4.4 Types of stresses

1. Tensile stress: When a body is applied with a load, which increase the length of the body as shown in fig 4.1 the stress devel oped in the body is called tensile stress. The corre sponding strain is called tensile strain.

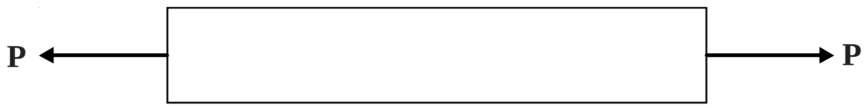


Fig.4.1

2. Compressive stress: When a body is applied with a load, which decrease the length of the body as shown in fig. 4.2, the stress developed in the body is called compressive stress. The corresponding strain is called compressive strain.

Fig.4.2

4.5 Mechanical properties of materials.

The study of mechanical properties of materials is necessary in the design of structural members. The important mechanical properties are

1. Elasticity: Any material when applied with loads undergoes deformation. When the load is removed, the body return back to its original shape and size. This is called elasticity.

2. Plasticity: Any material when loaded, shows elastic property upto a certain limit. Beyond this, the material does not get its original dimensions on removal of load. This is called plasticity.

3. Hardness: The hardness of a material is the resistance which it can offer for penetration, scratching, abrasion. Diamond, cast iron, etc. have relatively greater hardness than soft materials like copper, lead, etc.

4. Toughness: The toughness of a material is the resistance which it can offer for twisting or bend ing. Steel is tougher than cast iron.

5. Malleability: It is the property of the material to be extended in all directions by hammering or rolling. Gold and silver have the greatest malleability while cast iron and Nickel have the least malleability.

6. Ductility: It is the property of the material which can be drawn out in the direction of the length. Gold and platinum have the highest ductility.

7. Brittleness: It is the property of the material which can't be drawn out in the direction of length. This is opposite of ductility. Brittle materials get fracture even under small loads.

8. Fatigue: It is the property of the material to withstand cyclically varying loads.

9. Creep: It is the property of the material by which a material in tension will continue to elongate even under a constant load. Generally concrete structures are subjected to creep at high temperatures.

10. Stiffness: It is the property of the material to offer resistance to the deflection due to loading on the beams.

4.6 Stress - Strain diagram for mild steel

Mild steel rods are tested for their yield strength in a laboratory using universal testing machine. A test specimen is mounted in the testing machine and a gradually increased axial tensile load is applied on it. As the load is icnreased, the length of the spcimen increases. This change in length is measured by an extensometer which is attached to the specimen.

The Stress and strain values are calculated. A graph is plotted with stress versus strain as shown in Fig. 4.3. In the elastic range, the strain is directly proportional to stress. The point A is called limit of proportionality. When the load is increased, the stress increases and proportionately strain increases.

At any stress upto point B, the specimen will return to its original dimensions upon removal of the load. Beyond point B, the specimen will not be able to return to its original dimensions on removal of load. A permanent deformation occurs when the material is stressed beyond this point. So point B is called Elastic limit.

With further increase of load, the stress increases slowly while strain increases rapidly. The point C on the curve, corresponding to this stress is called yield point. Beyond C, the material becomes a little hardened, and the stress again increases with strain - until the maximum load is reached. The stress at D is called the ultimate stress. The stress falls from D to E and the specimen fails at E. The stress at E is called breaking stress.

A=Limit of proportionality

B=Elastic limit

C=yield point

D=Ultimate stress

E=Breaking Stress

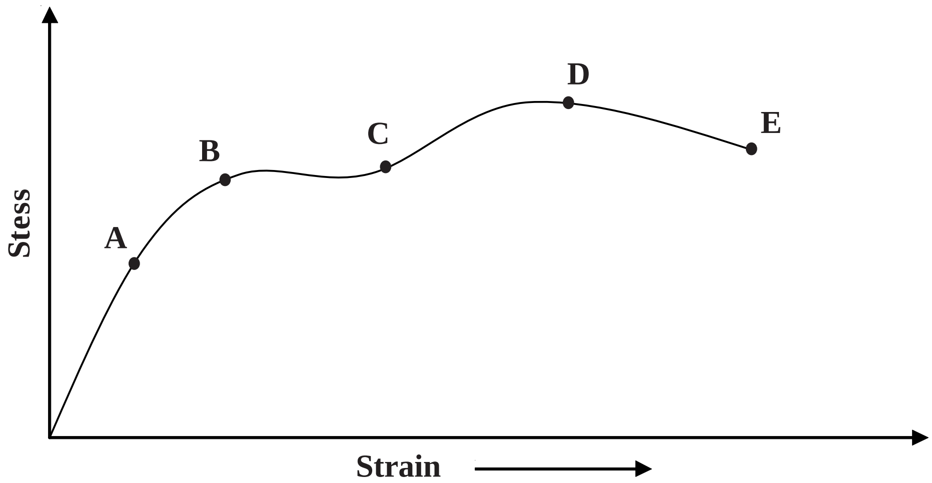


Fig.4.3

$$\text{Ultimate strength} = \frac{\text{maximum load}}{\text{original area of cross-section}}$$

Working stress :

The main aim of designing structural member is to prevent failure under given loads. So the members are designed in such a way that the stress on it is less than the maximum permissible stress, which is called the working stress.

Factor of safety :

In designining the structural members the working stress should be taken below the elastic limit. Working stress is calculated by dividing the ultimate stress with factor of safety.

4.7 Hooke's Law

It states that when a material is loaded, within elastic limit, stress is proportional to the strain.

$$\text{i.e. } \frac{\text{Stress}}{\text{Strain}} = \text{E} = \text{Constant}$$

Hooke's Law is valid for tension as well as compression.

4.8 Modulus of Elasticity (or Young's modulus):

A material is loaded, within elastic limit the ratio of stress to the corresponding strain is constant. This constant is called young's modulus or modulus of Elasticity, and is denoted by E.

$$\therefore \frac{f}{e} = E$$

Where f = Stress
e = strain
E = young's modulus or modulus of Elasticity.

4.9 Deformation of a body under axial load

Consider a body of uniform cross section A and length l, is subjected to an axial load p. The deformation $\delta\ell$ due to load p is given as stress

$$f = \frac{P}{A} \quad and\ strain\ e = \frac{\delta \ell}{\ell}$$

$$\therefore E = \frac{Stress}{Strain} = \frac{P \ell}{A \delta \ell}$$

$$Deformation, \quad \delta \ell = \frac{P\ell}{AE}$$

Table 4.1

The Values of E for different materials are given below

S.No.	**Material**	**Young's modulus (E) Kn/mm^2**
1	Mild steel	200 to 220
2	Cast Iron	100 to 120
3	Aluminium	70 to 80
4.	Copper	110 to 120
5.	Concrete	12 to 16
6.	Wrought Iron	180 to 200
7.	Timber	10 to 15

4.10. Units

In the SI units force is expressed in newtons. The kilo newton(KN) is equal to 1000 newtons. In the MKS units force is expressed in kg. Stress is expressed in various forms like newton /mm^2, newton / m^2, kg/ cm^2, kg/m^2

1 N/ metre^2 = 10 $^{-6}$ N/ mm^2
1 N/mm^2 = 10^6 N/ metre^2 = 1 mega newton / metre ^{2}
= 10^6 pa (pascal) = 1 mpa

strain is a dimensionless quantity.
The units for modulus of elasticity are same as stress.

Example 4.1:

A steel rod 25 x 10 mm and 200mm long is subjected to an axial pull of 55 KN. Find (i) the intensity of stress (ii) the strain (iii) elongation. Take E = 100 KN / mm^2.

Solution :

Area of cross - section = 25 x 10 = 250 mm^2

Load on rod p = 55 KN = 55000 N

i) Intensity of stress = $f = \frac{P}{A} = \frac{55000}{250} = 220\, N/mm^2$

ii) Strain = $e = \frac{f}{E} = \frac{220}{100 \times 10^3} = 0.0022$

iii) Elongation $(\delta \ell)$ = strain x original length = 0.0022 X 200 = 0.44 mm

Example 4.2 :

A steel rod 25 mm in diamater and 1 metre long has to carry an axial pull of 60 KN. Find (i) the intensity of stress (ii) the strain and (iii) elongation. Take E = 100 KN / mm^2.

Solution :

Length of rod = 1m

Pull = P = 60 KN = 60000 N

Area of rod $A = \frac{\Pi}{4}(25)^2 = 490.9 mm^2$

i) Intensity of stress $f = \frac{P}{A} = \frac{60000}{490.9} = 122.22\, N/mm^2$

ii) Strain $= e = \frac{f}{E} = \frac{122.22}{100 \times 10^3} = 1.22 \times 10^{-3}$

iii)Elongation = $\delta \ell = \frac{P\ell}{AE} = \frac{60000 \times 1000}{490.9 \times 100 \times 1000} = 1.22 mm$

Example 4.3

A circular steel rod has to carry a load of 25 KN. If the allowable stress in the rod is 100 N/mm^2. Find the dia-of rod required.

Solution :

Load on rod P = 25 KN = 25000 N
Allowable stress F = 100 N / mm ^{2}

Area of cross - section required $A = \frac{P}{F} = \frac{25000}{100} = 250\, mm^2$

If d is the dia. of rod,

$$A = \frac{\pi}{4} d^2, \; d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 250}{\pi}} = 318.47\, mm$$

A dia. of 319 mm will be taken.

Example 4.4 :

A steel rod of 25mm in dia and 200mm long extends 0.25mm under a tensile load of 40KN. Calculate stress, strain and E. (I.P.E. March 2007)

Solution:

Length of rod = ℓ = 200 mm

Area of cross- section of rod A = $\frac{\pi}{4}(25)^2$ = 490.60 mm^2

Load P = 40 KN = 40,000N

Elongation $\delta \ell = 0.25$ mm

Stress $= f = \frac{P}{A} = \frac{40000}{490.62} = 81.52\, N/mm^2$

Strain $= e = \frac{\delta \ell}{\ell} = \frac{0.25}{200} = 1.25 \times 10^{-3}$

$$\therefore \text{ Young's modulus} = E = \frac{f}{e} = \frac{81.52}{1.25 \times 10^{-3}} = 65216 \ N/mm^2$$

Example 4.5:

A hollow cast iron pipe has to carry an axial load of 360 Kn. Its external diamter is 22 cm, ultimate stress in the pipe is 600 N/ mm^2 and factor of safety is 3. Find the internal diamter of the pipe.

Solution:

Axial load	=	360 KN = 36000N
External diamter	=	22 cm = 220 mm
Internal diamter	=	d
Ultimate stress	=	600 N / mm^2
Factor of safety	=	3

$$\therefore \quad \text{Allowable stress } f = \frac{\text{Ultimate Stress}}{\text{Factor of Safety}}$$

$$= \frac{600}{3} = 200 \ N/mm^2$$

$$\text{Area of cross-section required} = A = \frac{Load}{Stress} = \frac{36000}{200}$$

$$= 180 \ mm^2$$

$$\therefore \frac{\pi}{4}(220^2 - d^2) = 180, \qquad 220^2 - d^2 = \frac{180 \times 4}{\pi} = 229.29$$

$$d^2 = 220^2 - 229.29$$

$$d = \sqrt{48170..71} = 219.47mm \cong 220mm$$

Example 4.6 :

A steel bar of uniform diameter 25 mm is subjected to axial forces as shown in Fig.4.4. Find the toal change in length of the bar. The $E_s = 200KN/mm^2$

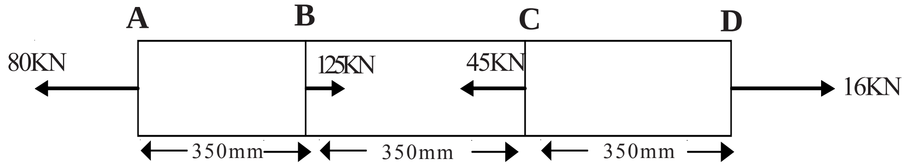
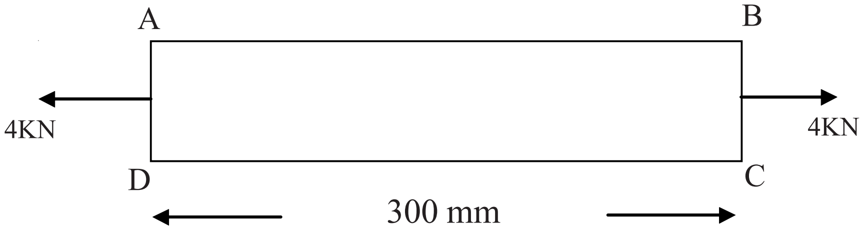


Fig.4.4

Solution: part AB:

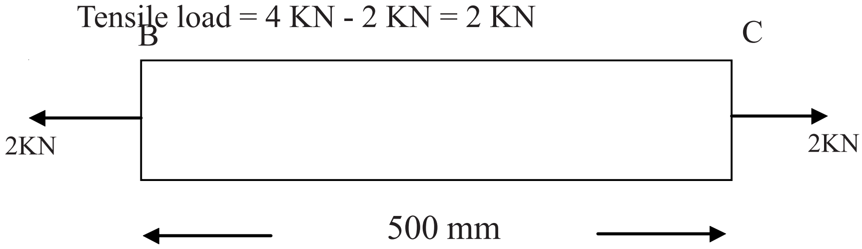


$$\therefore \text{ Change in length } = \frac{p_1 \ell_1}{AE}$$

$$\delta \ell_1 = \frac{4 \times 300}{\frac{\pi}{4}(25)^2 \times 200} = 0.0122 \, (Extension)$$

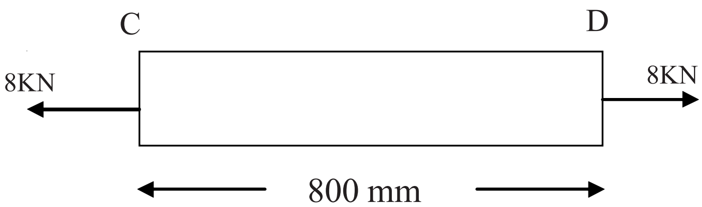
Part BC :

Tensile load = 4 KN - 2 KN = 2 KN



$$\delta\ell_2 = \frac{P_2 \ell_2}{AE}$$

$$\therefore \text{Change in length} = \frac{2 \times 500}{\frac{\pi}{4}(25)^2 \times 200} = 0.010\text{mm (Extension)}$$



Part CD :

Tensile load = 8 KN

$$\therefore \text{Change in length} = \frac{P_3 \ell_3}{AE} = \frac{8 \times 800}{\frac{\pi}{4}(25)^2 \times 200} = 0.016\text{mm (Extension)}$$

$$\therefore \text{Total extension in the bar} = \delta\ell_1 + \delta\ell_2 + \delta\ell_3 = 0.0122 + 0.010 + 0.016$$

$$= 0.0385\, mm$$

Example 4.7

A circular brass rod is to carry a tensile load of 10 KN. If the average stress is not to exceed 50 N/mm^2, What diameter of rod is required (IPE Sept, 1995)

Solution:

load = 10 KN = 10000 N

Average stress = 50 N/ mm^2

Diameter of rod = d mm

$$\text{Stress} = \frac{Load}{Area}$$

$$\text{Area} = \frac{Load}{stress} = \frac{10000}{50} = 200\, mm^2$$

$$\frac{\pi}{4} d^2 = 200$$

$$d^2 = \frac{200 \times 4}{\pi} = 254.77$$

$$d = \sqrt{254.77} = 15.96 \cong 16 mm$$

4.11 Shear stress and shear strain

Consider a rectangular block of length L and height H fixed at the bottom face AB. Let a force P be applied tangentially on the face AB. As a result of the force P, the different horizontal layers try to slide over one another and the block be distorted.

From ABCD to ABC_1D_1 as shown in Fig.

For the equilibrium of the block, it should offer a tangential reaction p equal and oppsite to the applied tangentail force P. The intensity of the shear resistance produced in the block is called as shear stress.

$$\therefore \text{Shear stress} = q = \frac{\text{Shear resistance}}{\text{shear area}}$$

The angular deformation ϕ in radians is called as shear strain.

$$\therefore \text{ Shear strain } = \phi = \frac{cc_1}{L}$$

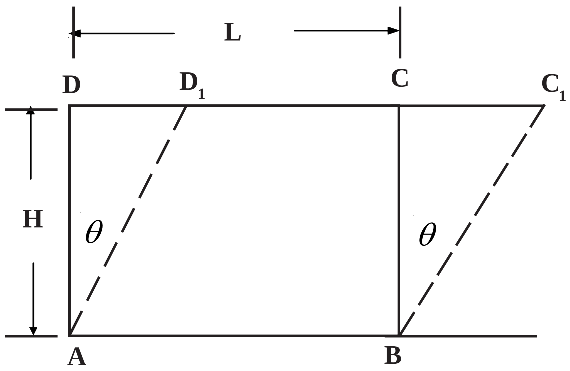


Fig.4.5

4.12 Shear modulus or modulus of Rigidity

When the shear deformation is within elastic limit, the ratio of the shear stress to the corresponding shear strain is called as shear modulus or modulus of Rigidity. Generally it is denoted by N,C, or G.

$$\therefore \text{Modulus of Rigidity} = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$= \frac{q}{\phi} = N$$

4.13 Longitudinal and Lateral Strain Poisson's Ratio

Consider a rectangular bar of length ℓ, width b, depth d be subjected to a tensile load P. The length of the bar along its longitudinal axis will increase while the lateral dimensions like width and depth will decrease. Let $\delta\ell$ is the increase in the length of the bar and $\delta b, \delta d$ be the decrease in width and depth.

$$\therefore \quad Longitudinal\ strain = \frac{\delta \ell}{\ell}$$

$$\text{Lateral Strain} = \frac{\delta b}{b} \ or \ \frac{\delta d}{d}$$

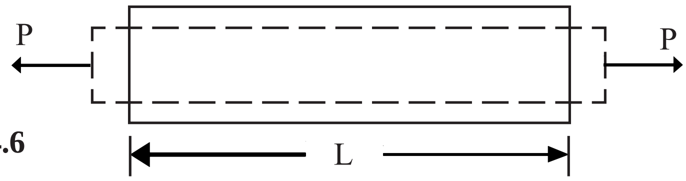


Fig.4.6

Within elastic limits, the ratio of the Lateral strain to the longitudinal strain is called as poisson's ratio and it is denoted by 1/m.

$$\therefore \quad \text{Poisson's ratio} = \frac{1}{m} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

4.14 Volumetric Strain

When a body is subjected to force, it undergoes changes in its dimensions which causes change in volume. The ratio of change in volume to original volume is called as volumetric strain. Generally this is denoted by e_v

$$\therefore Volumetric\ Strain = \frac{Change\ in\ Volume}{orignal\ Volume}$$

$$e_v = \frac{\delta v}{v} = e\left(1 - \frac{2}{m}\right)$$

Where e = strain

$1/m$ = poisson's ratio

4.15 Bulk Modulus

When a body is subjected to three equal mutually perpendicular stresses, the ratio of direct stress to the corresponding volumetric strain is called as Bulk modulus.

$$\therefore \quad Bulk\ Modulus = K = \frac{Direct\ stress}{Volumetric\ Strain} = \frac{f}{e_v}$$

Assume that, the three stresses cause volumetric change, total volumetric strain

$$e_v = 3e\left(1 - \frac{2}{m}\right)$$

$$\therefore \qquad K = \frac{f}{e_v} = \frac{f}{3e\left(1-\frac{2}{m}\right)} = \frac{E}{3\left(1-\frac{2}{m}\right)} \qquad \left(\because E = \frac{f}{e}\right)$$

4.16 Relation between Elastic constants:

The Elastic constants of a material are

E = young's modulus 1/ m = poisson ratio

N = Rigidity modulus K = Bulk modulus

The relationship between the Elastic constants are given as

$$E = 2N\left(1 + \frac{1}{m}\right)$$

$$E = 3K\left(1 - \frac{2}{m}\right)$$

$$E = \frac{9KN}{3K + N}$$

Example 4.8:

A round bar 20 mm diameter and 200 mm long was tested in tension. The increase in length was found to be 1.5 mm while the decrease in its diameter was 0.03 mm. Calculate the longitudinal strain, lateral strain and poisson's ratio. (IPE, March '96)

Solution :

Diameter of the bar = d = 20 mm

length of bar = L = 200 mm

Increase in length = δ L = 1.5 mm

Decrease in diameter = δ d = 0.03mm

$$\text{Longitudinal Strain} = \frac{\delta L}{L} = \frac{1.5}{200} = 7.5 \times 10^{-3}$$

Lateral Strain = $\frac{\delta d}{d} = \frac{0.03}{20} = 1.5 \text{ x } 10^{-3}$

Poisson's ratio = $\frac{1}{m} = \frac{Lateral\ Strain}{Longitudinal\ Strain} = \frac{1.5\times10^{-3}}{7.5\times10^{-3}} = 0.20$

Example 4.9:

A steel rod 20 mm diameter, 500 mm long is subjected to an axial pull of 160 KN. Find the change in length, diameter, change in volume Take E = $2 \text{ x } 10^5$ N/ mm^2 and poisson's ratio is 0.3.

Solution :

Diameter of steel rod = 20mm

Length of steel rod = 500mm

Area of cross section of steel plate = $\frac{\pi}{4}(20)^2$ =314 mm^2

$$\text{Stress}=\frac{\text{Load}}{\text{Area of crosssection}}=\frac{160\times10^3}{314}=0.5\times10^3\ \text{N/mm}^2$$

$$\text{Strain}=\frac{\text{Stress}}{\text{E}}=\frac{0.5\times10^3}{2\times10^5}=0.25\times10^{-2}$$

$$\therefore \frac{\delta\ell}{\ell}=0.25\times10^{-2}$$

change in length ($\delta\ell$) = $0.25 \text{ x } 10^{-2} \text{ x } 500 = 1.25$ mm

Lateral strain = Poisson's Ratio X Linear strain

$$\frac{\delta d}{d} = 0.3\times0.25\times10^{-2} = 0.075\times10^{-2}$$

Change in diameter (δd) = $0.075 \times 10^{-2} \times 20 = 1.5 \text{ X } 10^{-2}$ mm

Change in volume (v) = $\frac{\pi}{4}d^2\times\ell=\frac{\pi}{4}\times20^2\times500 = 157000$ mm^3

Volumetric strain $e_v = e\left(1-\frac{2}{m}\right)$

= $0.25\times10^{-2}\left[1-2\times0.3\right] = 0.1 \times 10^{-2}$

$\therefore$ Change in volume $\delta v = \text{v} \times e_v = 157000 \times 0.1 \times 10^{-2} = 157$ mm^3

Example 4.10 The young's and Bulk modulus of material are given as 2 x 10^5 N/ mm^2 and 1.6 x 10^5 N/mm^2. Find rigidity modulus and poisson's ratio.

Solution :

Young's Modulus E= 2 x 10^5 N/mm^2
Bulk Modulus K = 1.6 X 10^5 N/mm^2
Using the relation between Elastic constants.

$$E = 3K\left(1 - \frac{2}{m}\right) \text{ and } E = 2N\left(1 + \frac{1}{m}\right)$$

$$\frac{1}{m} = \frac{1}{2}\left(1 - \frac{E}{3K}\right) = \frac{1}{2}\left(1 - \frac{2 \times 10^5}{3 \times 1.6 \times 10^5}\right) = 0.29$$

$$\text{Rigidity Modulus N} = \frac{E}{2\left(1 + \frac{1}{m}\right)}$$

$$= \frac{2 \times 10^5}{2(1 + 0.29)} = 0.77 \times 10^5 \; N/mm^2$$

EXERCISE 4.1

1. A steel bar 5.6 m long and 25mm diameter is subjected to an axial pull of 200N. The elongation in the bar is 0.16mm. Determine the stress and strain in the bar also find Young's Modulus of the bar.

2. A circular concrete pillar has to carry a compressive load of 1650 KN. The permissible stress in the pillar should not exceed 72 N/mm^2. Find the diameter of the pillar?

3. A steel rod of diametre 25 mm has to carry system of loads as shown in figure.4.7. Find the total change in the length of the rod. Take E = 200 KN/ mm^2

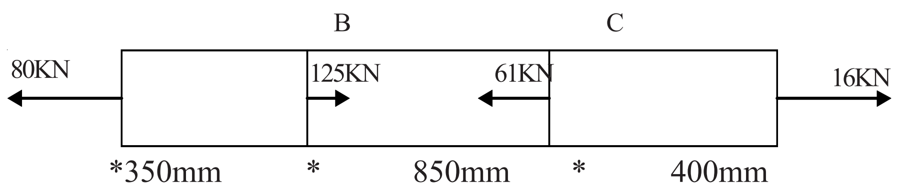


Fig. 4.7

4. A metal bar of cross section 25mm x 12.5mm and of length 1.6 m is subjected to an axial pull of 120 KN. Find the changes in the dimensions of the bar. Take $E = 2 \times 10^5$ N/mm^2, poisson's ratio = 1/3.

5. A bar of diameter 30mm and length 450 mm has to carry a tensile load of 170 KN. The change in the length of bar is 0.32mm and the reduction in the diameter is 0.025mm. Calculate (1)Young's modulus (ii) poisson's ratio (iii) change in volume.

6. The following readings were observed in a tension test on a metal bar:

original diameter	=	22mm
Gauge length	=	210 mm
Load	=	18.6KN
Change in length	=	0.024mm
Change in diameter	=	0.00018mm

Calculate the values of Elastic constants. Also find the change in volume

7. A hollow metal pipe 4 m long has 200 mm outer diameter and 10mm thickness. It has to support a compressive load of 400 KN. Calculate the decrease in length of the pipe. Take E = 100KN/mm^2

8. A steel plate of length 4.6m, width 100mm and thickness 30mm is subjected to an axial pull of 24 KN. Calculate the changes in length, Width, thickness and volume of the steel plate. Take E = 210KN/mm^2, 1/m = 0.29

4.17 Composite sections

A composite section consists of two or more different materials rigidly connected together. The load applied on the member will be shared by each member of the section.

Consider a composite section made of different materials as shown in fig.4.8

A_1, A_2 are the areas of individual members and their Modulus of Elasticity are E_1,E_2 respectively. P is the load applied on the section.

Let P_1 , P_2 are the loads shared by individual members.

$P = P_1 + P_2$ -- (1)

Let f_1 , f_2 are the stresses in the individual members,

Then $P = f_1A_1 + f_2 A_2$ -- (2)

Since the members are rigidly connected, the length of all members of the composite section is the same, the deformation of the individual member will be same. The strain in each member will be same.

Let the strains in the individual members are e_1, e_2

$\therefore$ $e_1 = e_2$ 3

or $\frac{f_1}{E_1} = \frac{f_2}{E_2}$ 4

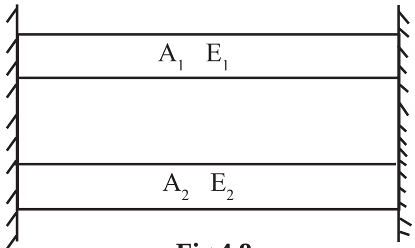


Fig 4.8

Example 4.11

A short reinforced concrete column is of square shape 400mm side. It is reinforced with 4 steel bars each 22 mm dia. placed near the corners. Assuming ratio of young's modulus of steel to young's modulus of concrete is 18. Calculate the stress in concrete and steel when the column carries an axial compressive load of 750KN. (IPE, Sept. 95)

Solution :

Area of column A = 400 x 400 = 160000 m^2

Area of steel $A_s = 4 \times \frac{\pi}{4} \times 22^2 = 1521\, mm^2$

Area of concrete $A_c = A - A_s = 160000 - 1521$
$= 158479\ mm^2$

Since the strains in individual members are same,

$$e_c = e_s$$

$$\frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$\therefore\ fc = \frac{E_c}{E_s} \times f_s = \frac{f_s}{18} \qquad \left(\because \frac{E_s}{E_c} = 18\right)$$

Load shared by steel (Ps) = $f_s . A_s = f_s$ x 1521

Load shared by concretes (Pc) = $f_c A_c = \frac{f_s}{18} \times 158479 = 8804.39\ f_s$

$P = P_s + P_c = 1521\ f_s + 8804.39\ f_s$

750 KN = 10325.39f_s

$\therefore$ Stress in Steel $f_s = \frac{750000}{10325.39} = 72.64 N/mm^2$

Stress in concrete $f_c = \frac{f_s}{18} = \frac{72.64}{18} = 4.035 N/mm^2$

Example 4.12:

A steel rod 25 mm dia and 600 mm long is fixed in a copper tube of the same internal diameter and thickness 12.5mm. The composite section has to carry a compressive load of 180KN. Determine the stresses developed in steel and copper. Take $E_s = 2.1 \times 10^5\ N/mm^2, E_c = 1.2 \times 10^5\ N/mm^2$.

Solution:

Area of the steel rod $A_s = \frac{\pi}{4} \times 25^2 = 490.78\ mm^2$

outer dia-of copper tube D = 25 + 2 thickness = 25 + 2 x 12.5 = 50mm

Area of copper tube $A_c = \frac{\pi}{4}\left(D^2 - d^2\right)$

$$= \frac{\pi}{4}\left(50^2 - 25^2\right)$$

$$= 1471.87 mm^2$$

In composite section, strain in steel rod (e_s) = Strain in copper tube (e_c) - (1)
Load shared by steel rod + load shared by copper tube = Load.

$f_s A_s + f_c A_c = P$ ----------- (2)

From Equation (1) $e_s = e_c \Rightarrow \frac{f_s}{E_s} = \frac{f_c}{E_c}$

$$f_s = \frac{E_s}{E_c} \times f_c = \frac{2.1 \times 10^5}{1.2 \times 10^5} f_c$$

$\therefore f_s = 1.75\ f_c$ --------(3)

Substitute Equation (3) in Equation (2)

$$\therefore 1.75\, f_c \times 490.78 + f_c\, 1471.87 = 180 \times 1000$$

Stress in copper (f_c) = $\dfrac{180000}{2330.73} = 77.22\, N/mm^2$

Stress in steel (f_s) = $1.75 \times 77.22 = 135.15\ N/mm^2$

Example 4.13

Two copper rods each of cross- sectional area of 300 mm^2 and a steel rod of cross sectional area 2200 mm^2 together support a rigid uniform beam weighing P newtons as shown in fig.4.9. The stresses in copper and steel not to exceed 160 N/mm^2 and 380 N/mm^2 respectively. Find the intensity of load. Assume $E_s/E_c = 2$.

Solution :

Since the strains are equal in different materials of composite section

$$e_s = e_c$$

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = \frac{E_s}{E_c} f_c = 2\, f_c$$

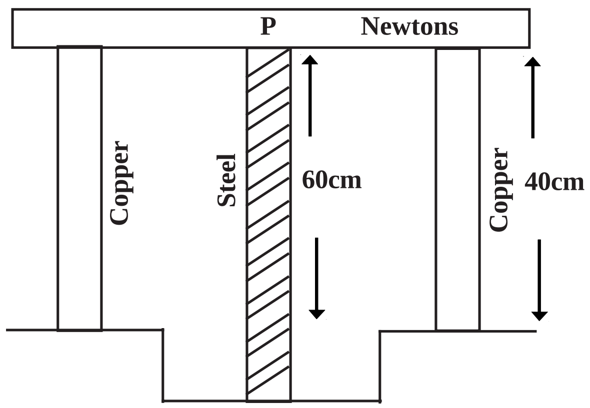


Fig.4.9

Area of copper rod A_c = 600 mm^2

Area of steel rod A_s = 2200 mm^2

Length of copper rod ℓ_c = 40cm = 400 mm

length of steel rod ℓ_s = 60 cm = 600 mm

Decrease in length of copper rod $(\delta\ell_c) = e_c \ell_c = \dfrac{f_c}{E_c} \times \ell_c$

Decrease in length of steel rod $\delta l_s = e_c l_c = \dfrac{f_c}{E_c} \times \ell_c$

Since the change in length of copper and steel is same

$$\therefore \frac{P_s}{E_s} \times \ell_s = \frac{f_c}{E_c} \times \ell_c = \frac{f_s}{E_s} \times \ell_s$$

$$f_s = \frac{E_s}{E_c} \times \frac{\ell_c}{\ell_s} \times f_c = 2 \times \frac{400}{600} \times f_c = \frac{4}{3} f_c$$

$$If\ f_s = 380\, N/mm^2\ then\ f_c = \frac{3}{4} \times f_s = \frac{3}{4} \times 380 = 285\, N/mm^2$$

This is more than the allowable stress($\because f_c = 160\, N/mm^2$)

If $f_c = 160\, N/mm^2$ then $f_c = \frac{4}{3} f_c = \frac{4}{3} \times 160 = 213.33\, N/mm^2$. This is less than the allowable stress 380 N/mm².

$$\therefore Load, p = f_s A_s + 2(f_c A_c) = 213.33 \times 2200 + 2(285 \times 600)$$

$$= 811326N = 811.32KN$$

4.18. Temperature stresses

When the temperature of a composite section is either increased or decreased there will be change in the dimensions of the body. If this change is prevented stress will be developed in the body.

The stress induced in a body due to prevention of deformation caused by change in temperature of the body is called temperature stress.

When the temperature of the bar is increased the bar expands and when decreased it contracts. If the body expansion is prevented due to rise in temperature compressive stresses will be developed in the body and if contraction is prevented in the body due to fall in temperature tensile stresses will be developed in the body. The amount of expansion or contraction of the material depends on its co-efficient of Linear expansion. It isdefined as the increase in length of a body per unit rise in temperature and is denoted by a Greek letter $\alpha(Alpha)$. In SI units the temperature is measured as kelvin (k) $[1^0 c = 273^0.15K]$

Consider a bar of length ℓ, co-efficient of linera expansion α and Young's modulus E, heated to a temperature of $\in$ Kelvin

Expansion in the bar = $\ell \alpha t$

(i) If both ends of the bar are fixed temperature strain $= \frac{\ell \alpha t}{\ell} = \alpha t$

(ii) Temperature stress = $E\alpha t$

If the ends yield by an amount e, expansion

prevented = free expansion - yield of ends

= $\ell \alpha t$ -e

Temperature strain $= \frac{\ell \alpha t - e}{\ell}$

Temperature stress = $E = \left(\frac{\ell \alpha \in - e}{\ell} \right)$

Example 4.14:

A stell rod of 20mm diameter and 3m long is connected to two grips at a temperature of 120^0c. Find the pull exerted when the temperature falls to 50^0C, if

a) The ends do not yield

b) The ends yield by 0.04 mm

Take E = 2 x $10^5 \frac{N}{mm^2}$ α = 12 x 10^{-6} / ^{0}C (IPE, March '96)

Solution:

Area of the rod $= \frac{\pi}{4} \times 20^2 = 314.16\, mm^2$

Contraction in the rod due to decrease in temperature

$= \ell \ \alpha \ t = 3000 \times 12 \times 10^{-6} \times \left(120^0 - 50^0\right)$

$= 2.52\, mm$

a) When the ends do not yield.

$$Temperature\ strain = \frac{Contraction\ prevented}{original\ length}$$

$$= \frac{2.52}{3000} = 0.84 \times 10^{-3}$$

Temperature stress = strain X E

= 0.84 x 10^{-3} 2 x 10^{5} = 168 N / mm^2

Load (or) pull exerted = Temperature stress x area

= 168 x 314 . 16 = 5.17 X 10^4 N

b) The ends yield by 0.04 mm, the contraction prevented = 2.52 - 0.04 = = 2.48 mm

$$Temperature\ strain = \frac{2.48}{3000} = 8.27 \times 10^{-4}$$

Temperature stress = strain x E

= 8.27 x 10^{-4} x 2 x 10^5 = 165 . 33 N/mm^2

Load (or) pull exterted = 165.33 x 314 .16 = 5.19 x 10^4 N

Example 4.15:

A steel rod of length 5m is at a temperature of 200 kelvin. Find the increase in length when the temperature is increased by 50 kelvin. Find the temperature stress in the bar when the free expansion is prevented $\alpha = 12 \times 10^{-6}$ per kelvin and E = 200 KN / mm^2

Solution :

length of steel rod = 5m = 5000 mm

Increase in temperature = 50 kelvin

Co-efficient of linear expansion (α) = 12 x 10^{-6} per kelvin

Increase in length of bar $(\delta\ell) = \ell\alpha\in$

= 12 x 10^{-6} x 50 x 5000 = 3mm

Temperature stress in the bar, when the exapansion is prevented = $\alpha \in E$

= 12 x 10^{-6} x 50 x 200 = 0.12 N/mm^2

EXERCISE4.2

1. A copper bar 40 mm diameter is inserted into a steel tube of 50 mm external diameter and 5mm thick. If the lenght of the arrangement is 3000 mm, find the stresses and expansion in copper and steel when it has to carry a load of 200 KN. Take $E_c = \frac{1}{2} E_s$

2. A steel rod 20mm dia and 30m long is placed in a brass tube of 30mm outer dia and 5mm thick. The composite section is subjected to a compressive load of 150KN. Find the stresses produced in steel and brass.

$E_s = 2.1 \times 10^5\ N/mm^2, E_b = 1 \times 10^5\ N/mm^2$

3. A bar of length 5m is rigidly held between two ends and heated through 600K. Calculate the temperature stress in the bar. Tak E = 2 x 10^5 N/mm^2 and α = 11 x 10^{-6} per kelvin.

4. A steel rod 25mm diameter and 8m long is fixed between two grips at a temperature of 160^0c. Find the pull exerted when the temperature is decreased to 80 ^{0} C

a) If the ends do not yield
b) If the ends yield by 1.1 mm

$Take\ E = 2 \times 10^5\ N/mm^2\ ,\ \alpha = 12 \times 10^{-6} / {}^0C$

5. Two parallel walls 15m apart are stayed together by a steel rod of 50mm diameter at a temperature of 120^0C. Determine the stresses in the rod when the temperature falls down to $70^0{}_{C,}$ if

i) the ends do not yield.
ii) the ends yield by 10mm

$Take\ E = 2 \times 10^5\ N/mm^2\ ,\ \alpha = 12.5 \times 10^{-6} / {}^0C$

5
SHEAR FORCE AND BENDING MOMENT

5.1 Introduction:

A beam is a horizontal member designed to support loads. These loads act at perpendicular to the axis of the beam. The loads cause bending in the beam. Beams are generally made with timber, steel, R.C.C. etc., and are provided to support floors, roofs, walls, etc.

5.2 Types of beams:

Based on the end conditions of the member the beams are classified as (fig. 5.1)

(a) Cantilever beam (b) simply supported beam
(c) over hanging beam (d) fixed beam (e) continuous beam.

a) Cantilever beam:

This is the beam in which one end is fixed and the other end is free.

b) Simply supported beam

This is the beam in which the beam is freely resting on two end supports. The distance between the face of the supports is called clear span and the centre distance of end supports is called effective span

c) Over hanging beam

The beam is simply supported at two points and extended at one or both ends beyond the supports.

d) Fixed beam

The beam is fixed at both ends.

e) Continuous beam

The beam which has more than two supports. The span between the supports may be equal or unequal.

5.3 Types of supports

Based on the nature of support and the support conditions, the beam supports are classified as: [Fig 5.2]

a) Simple supports b) Pin or hinged supports
c) Roller supports d) Fixed supports

a) Simple supports:

A simple support offers resistance to vertical movement only. It will not offer any resistance to other movements like horizontal movement or rotation.

b) Pin or Hinged supports

A pin or hinged support offers resistance to vertical and horizontal movement. It will not offer resistance to rotation.

c) Roller supports:

A roller support offers resistance to vertical movement only. It is similar to simple support.

d) Fixed supports:

A fixed support offers resistance to lateral movement and rotation. It holds the beam rigidly. It offers vertical reaction, horizontal reaction and moment reaction.

5.4 Types of loads:

A beam may be subjected to the following types of loads: [Fig.5.3]

a) point or concentrated load
b) Uniformly distributed load
c) Uniformly varying load.

a) Point or concentrated load

A load acting at a point on a beam is known as point or concentrated load.

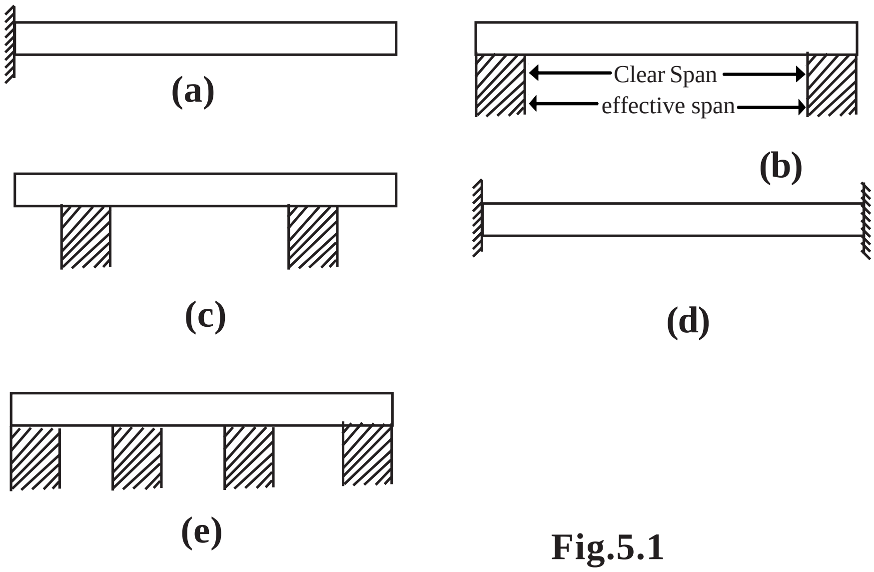


Fig.5.1

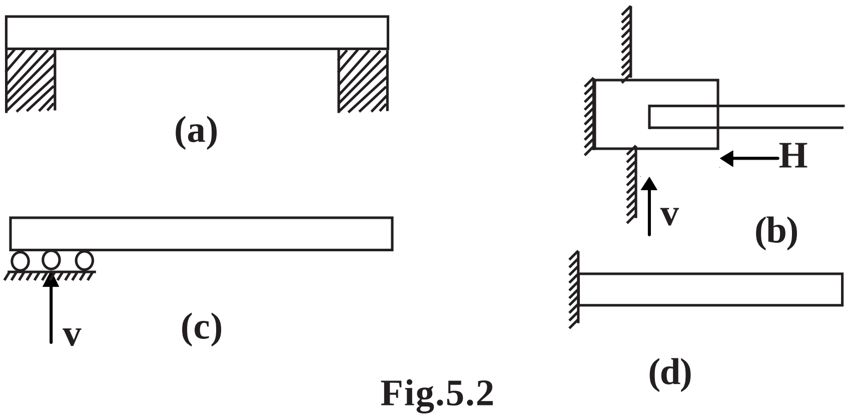


Fig.5.2

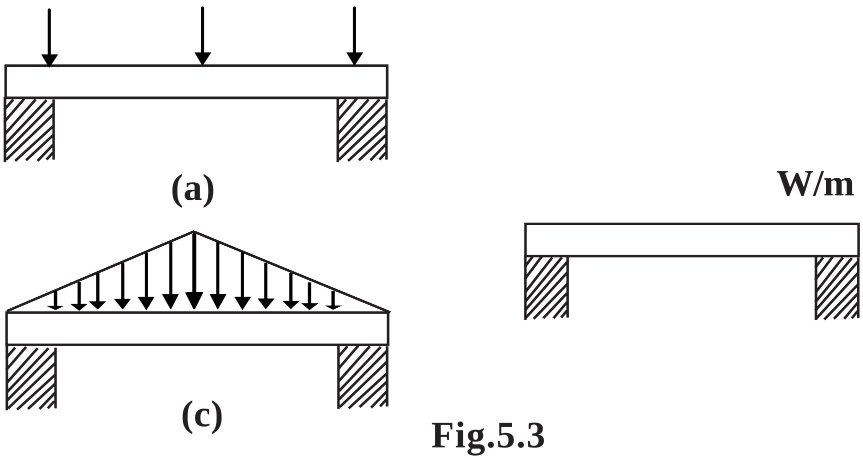


Fig.5.3

b) Uniformly distributed load.

A load which is uniformly spread over a beam such that each unit lenth is having same load is known as uniformly distributed load. It can also written as U.D.L.

c) Uniformly varying Load

A load which varies uniformly on each unit length is known as uniformly varying load.

5.5 Shear force and Bending moment

Shear force at any section of a beam may be defined as the algebraic sum of vertical forces to the right or left of the section.

Bending moment at any section of a beam may be defined as the algebraic sum of the moments of the forces, to the right or left of the section.

5.6 Sign conventions

Different sign conventions are used in different books regarding shear force and bending moment at a section. In this book, the following sign conventions are used.

a) Shear force

A shear force which is right upward or left downward at a section is said to be positive. Similarly a shear force which is left upward or right downward at a section is said to be negative.

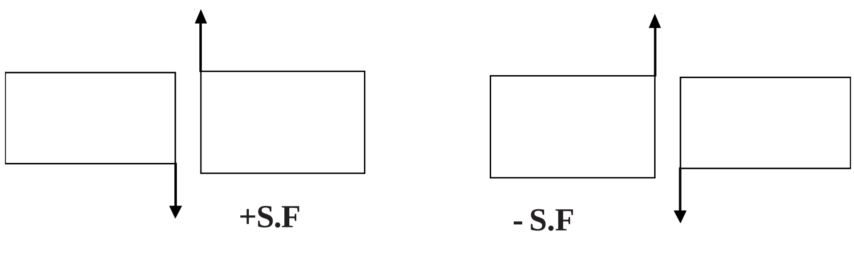
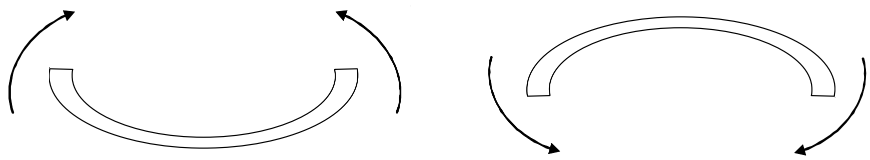


Fig. 5.4

b) Bending moment

If a beam is subjected to clockwise moment to left hand side and anticlockwise moment to the right hand side, it causes sagging of the beam. Sagging moment is positive bending moment.

If a beam is subjected to anticlockwise moment to left hand side and clockwise moment to right hand side, it causes hogging of the beam. Hogging moment is negative bending moment.



+B.M **- B.M**

Fig. 5.5

5.7 Shear force and bending moment diagrams

The shear force and bending moment values vary from section to section on the beam. They depends on the type of loading on the beam. These values may be calculated and plotted along the base line of the beam.

In shear force diagram the values of shear force are plotted as ordinates along the base line. The base line represents the axis of the beam.

This graphical representation of variations of shear forces is known as shear force diagram (S.F.D.)

In bending moment diagram the values of moments are plotted as ordinates along the base line. The base line represents the axis of the beam. The graphical representation of variations of bending moments is known as bending moment diagram (B.M.D.)

While drawing the B.M. or S.F. diagrams, positive values are generally plotted above the base line and negative values below it. These diagrams are very useful in designing of beams.

5.8. S.F. and B.M. diagrams for cantilever beams

1) Cantilever with a point load at the free end

Let a cantilever beam AB of length L carrying a point or concentrated load W at free end. Consider a section XX at a distance of 'x' from free end.

Shear force at the section xx F_x = - w(-ve due to right downward)

This value is constant for all sections of the cantilever. Hence the S.F.D. is a line parallel to the base.

Bending moment at the section XX
M_x = - Wx (- ve due to hogging)

Here the bending moment depends on x.

At x = 0, Bending moment (M_0) = 0

x = L, Bending moment (M_L) = -WL

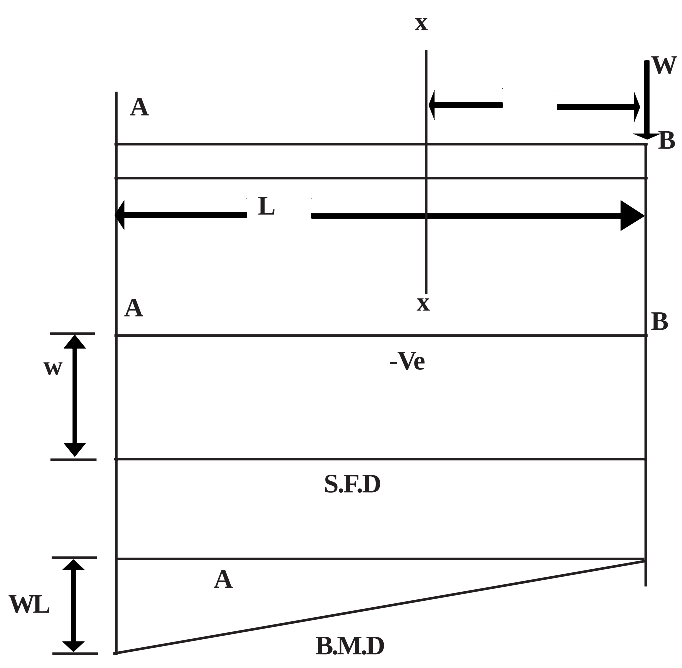


Fig.5.6

The bending moment will change uniformly from free and to fixed end as an inclined straight line.

Example 5.1:

A cantilever beam of span 5m is subjected to a point load of 10KN at a distance of 2m from free end. Draw shear force and bending moment diagram.

Solution :

Shear Force:

F_B=0

F_C=-10KN (-Ve due to down ward)

F_A=-10KN

Bending moment:

M_B=0

M_C=0

M_A=-10x(5-2)=-30KN-m

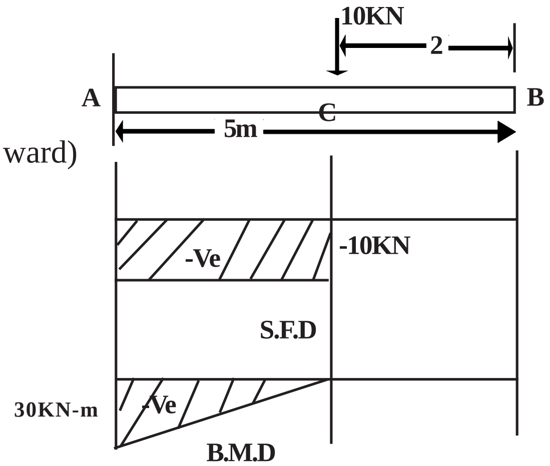


Fig. 5.7

Example 5.2:

A Cantilever 8m long is subjected to concentrated loads of 10,20,30 and 40KN at a distance of 2,3,5 and 7m from free end. Draw shear force and bending moment diagram. State the position of maximum shear force and bending moment (I.P.E,march 1995).

Solution :

Shear force

F_B = 0

F_C = -10 KN

F_D = -10 - 20 = - 30 KN

F_E = - 10 -20-30 = - 60 KN

F_F = -10 - 20-30-40 = - 100 KN

F_A = -10 -20-30-40 = - 100 KN

Bending moment :

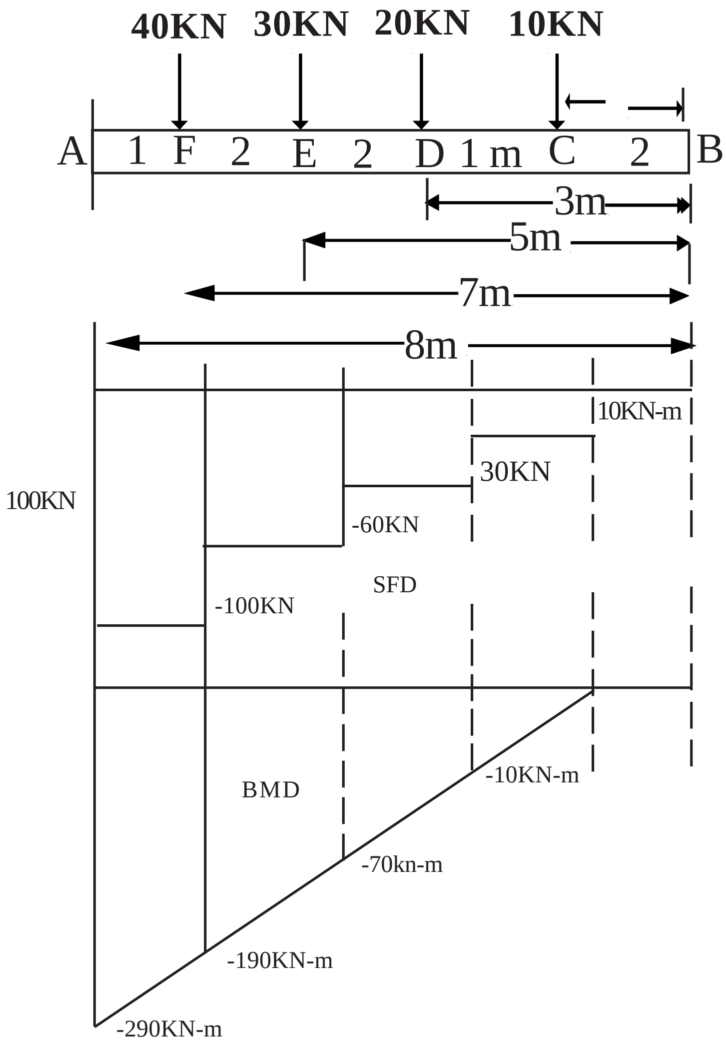


Fig. 5.8

$$M_B = 0$$

$$M_C = 0$$

$$M_D = -10 \times (3-2) = -10KN - m$$

$$M_E = -10(5-2) - 20(5-3) - 70Kn - m$$

$$M_F = -10(7-2) - 20(7-3) - 30(7-5) = -190Kn - m$$

$$M_A = -10(8-2) - 20(8-3) - 30(8-5) - 40(8-7)$$

$$= -290KN - m$$

Maximum shear force is at A, F_{max} = - 100 KN
Maximum Bending moment is at A, M_{max} = - 290 Kn-m

(2) Contilever with uniformly distributed load

Let a cantilever beam AB of length L, carrying a uniformly distributed load w per unit length as shown in Fig. 5.4. Consider a section XX at a distance of 'x' from free end.

Shear force at the section XX

F_X = - wx (-ve due to downward load)

At x = 0 , F_0 = 0

At X = L, F_L = - WL

The shear force diagram will be
an inclined straight line, 0 at B and WL at A.

Bending moment at XX,

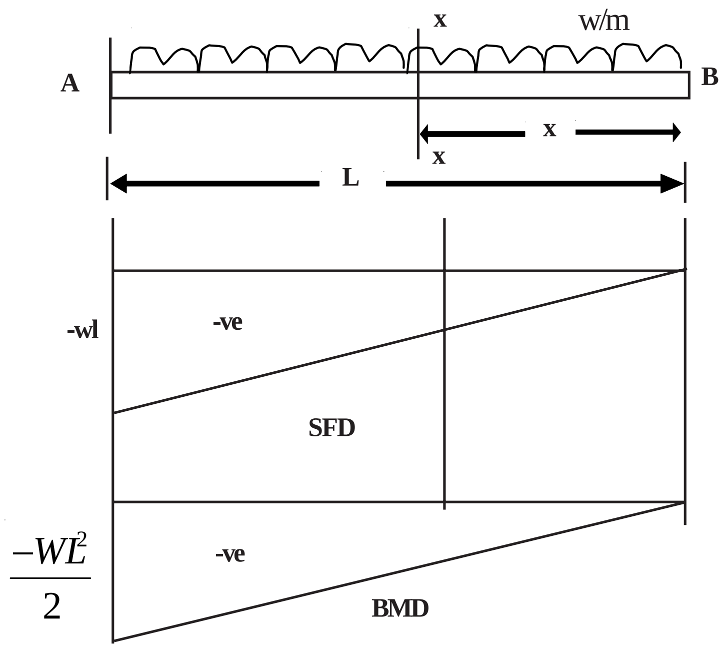


Fig.5.9

$$M_X = -WX.\frac{x}{2} = \frac{-Wx^2}{2}$$

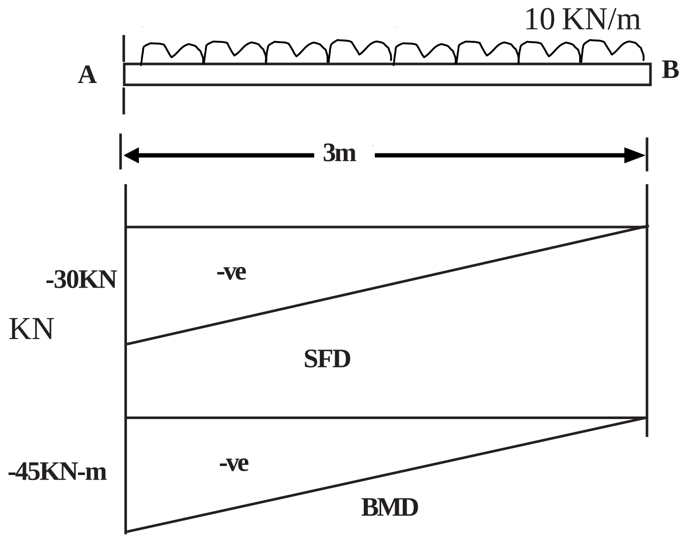
$$At\ x = 0, M_B = 0$$

$$At\ x = L, M_A = \frac{-WL^2}{2}$$

The bending moment diagram is a parabola with a value of 0 at B and $\frac{-W\ell^2}{2}$ at the fixed end A.

Example 5.3:

A cantilever 3 m long is loaded with a U.D.L. of 10 KN/m for entire span. Draw SFD and BMD.



Solution:

Shear Force

$F_B = 0$

$F_A = -10 \times 3 = -30$ KN

Bending moment :

$M_B = 0$

$$M_A = -10 \times 3 \times \frac{3}{2} = -45\ KN - m$$

Fig.5.5

Example 5.4:

A cantilever 5 m long is loaded with a u.d.l load of 20 Kn/m for 1 m length from free end and concentrated loads of 8 KN and 12 KN at distances of 2m and 3m from free end. Draw SFD and BMD.

Solution :

Shear force

$F_B = 0$
F_C = -20 x 1= - 20 KN
F_D = -20 x 1 - 8 = - 28 KN
F_E = -20 x 1 - 8 -12 = -40 KN
F_A = - 20 x 1 - 8 - 12 = - 40 KN

Bending moment :

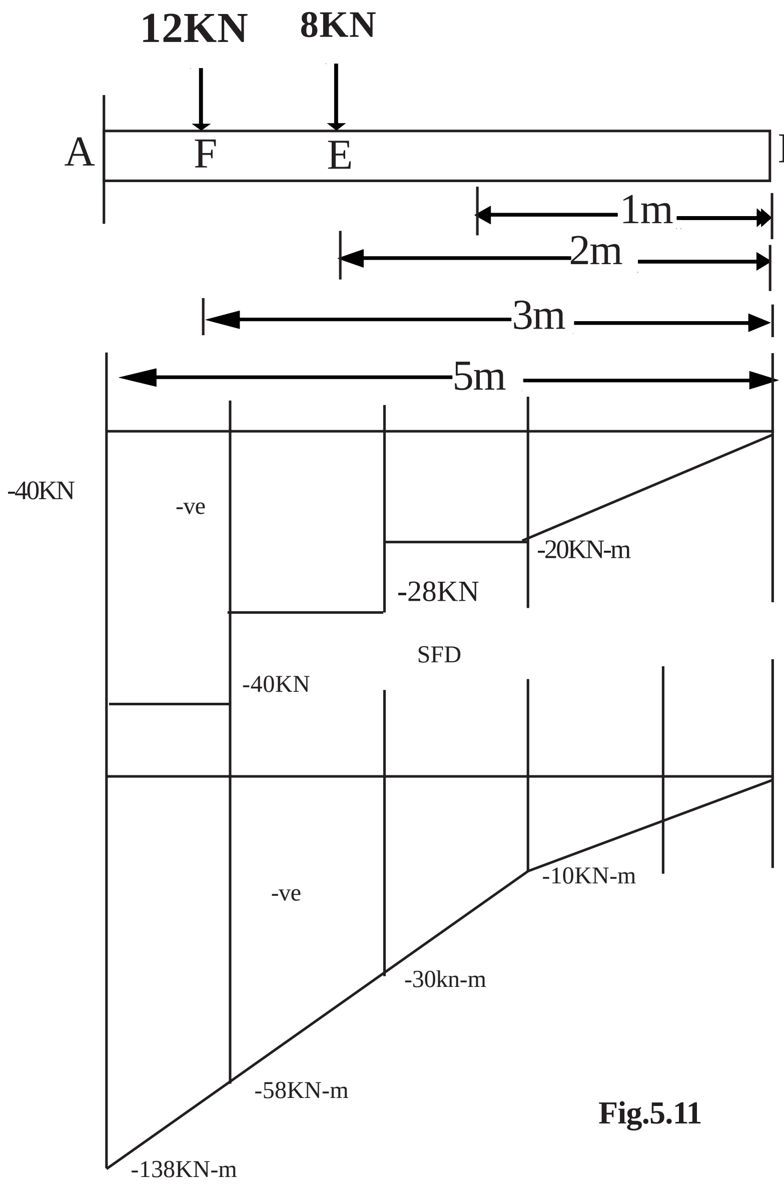


Fig.5.11

B.M.D

$$M_B = 0$$

$$M_C = -20 \times 1 \times \frac{1}{2} = -10KN - m$$

$$M_D = -(20 \times 1)\left(\frac{1}{2} + 1\right) = -30KN - m$$

$$M_E = -(20 \times 1)\left(\frac{1}{2} + (3-1)\right) - 8(3-2)$$

$$= -50 - 8 = -58\,KN - m$$

$$M_A = -(20 \times 1)\left[\frac{1}{2} + (5-1)\right] - 8(5-2) - 12(5-3)$$

$$= -90 - 24 - 24 = -138\,KN - m$$

EXERCISE - 5.1

1. A cantilever beam 5m long carries point loads of 2,3, 5 KN at 1, 3, 5 m from fixed end. Draw SFD and BMD.

2. A cantilever beam 6 m long carries a point load of 10 KN at 2 m from free end. Draw SFD and BMD.

3. A cantilever beam 4m long carries an u.d.l. of 2 Kn/m over the half length from free end. Draw SFD and BMD.

4. A cantilever beam of 6m is subjected to a uniformly distributed load of 20 N/m and point loads of 100 N and 200N at distances of 2m and 4 m from fixed end. Draw shear force and bending moment diagrams. Show the position where maximum shear force and Bending moment occurs. (I.P.E. March, 1996)

5. A cantilever AB is 6m long fixed at end A carries point loads of 10KN, 8KN, 12KN at 2m, 5m, 6m from fixed end. Draw SFD and BMD.
(I.P.E. March 2007)

5.9 S.F. and B.M. Diagrams for simply supported beams

1) Simply supported beam with point loads.

Let a simply supported beam (S.S.B.) AB of length L carrying a point load W at its centre.

Let R_A and R_B are the reactions of supports A and B

Taking moments about A,

$$R_B \times L = W \times L/2$$

$$\therefore R_B = \frac{W}{2}$$

$$R_A = w - R_B = w - \frac{w}{2} = \frac{w}{2}$$

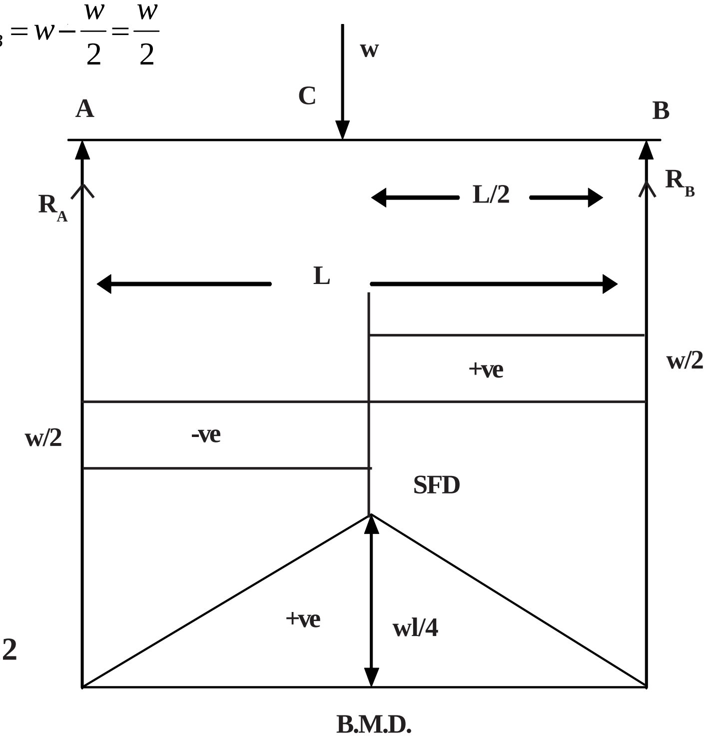


Fig. 5.12

Shear Force :

Shear force at any section between B and C is constant and is equal to + $\frac{w}{2}$ (upward positive). Shear force at any section between A and C is also constant and is equal to - $\frac{w}{2}$ (unbalanced vertical force $\frac{w}{2} - w = -\frac{w}{2}$)

Bending Moment:

The bending moment at free ends is zero. It increases by a straight line and is maximum
at the centre where S.F. bending of mement at c changes sign

$$M_C = \frac{w}{2} \times \frac{L}{2} = + \frac{WL}{4} \left(+ ve\ due\ to\ sagging \right)$$

Example 5.5

A simply supported beam 4m long is subjected to two point loads of 2 KN and 4 KN each at distances of 1.5 m and 3m from the left end. Draw S.F.D. and B.M.D.

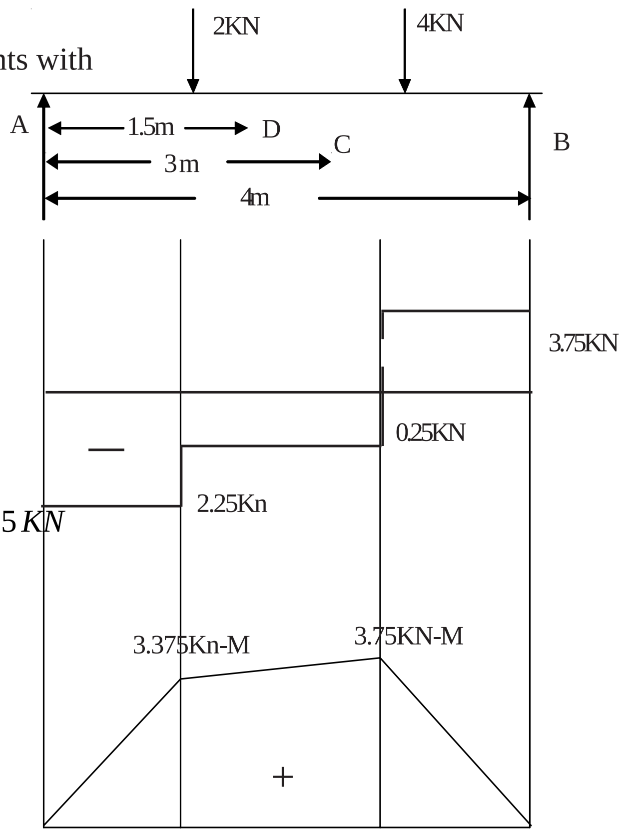
Solution:

Equating clock-wise moments with anti-clockwise moments
Taking moments about A,

$$R_B \times 4 = 2 \times 1.5 + 4 \times 3$$

$$R_B = \frac{15}{4} = 3.75\, KN$$

$$R_A = (2 + 4) - 3.75 = 2.25\, KN$$



Shear Force:

$$F_B = shear\ \ force\, at\ B = +3.75\, KN$$

$$F_C = +3.75 - 4 = -0.25\, KN$$

$$F_D = -0.25 - 2 = -2.25\, KN$$

$$F_A = -2.25 KN$$

Bending Moment:

$$M_A = M_B = 0$$

$$M_C = 3.75 \times 1 = 3.75\, KN - m$$

$$M_D = 3.75 \times (4 - 1.5) - 4 \times (3 - 1.5) = 3.375\, KN - m$$

Fig. 5.13

(2) Simply supported beam with u.d.l.

Let AB is a simply supported beam of length L is subjected to u.d.l. of W per unit run,

Let R_A , R_B are the reactions at supports A & B.

Taking moments about A,

$$R_B \times L = W \times L \times \frac{L}{2} = \frac{WL^2}{2}$$

$$R_B = \frac{WL}{2}$$

$$R_A = W.L - \frac{WL}{2} = \frac{WL}{2}$$

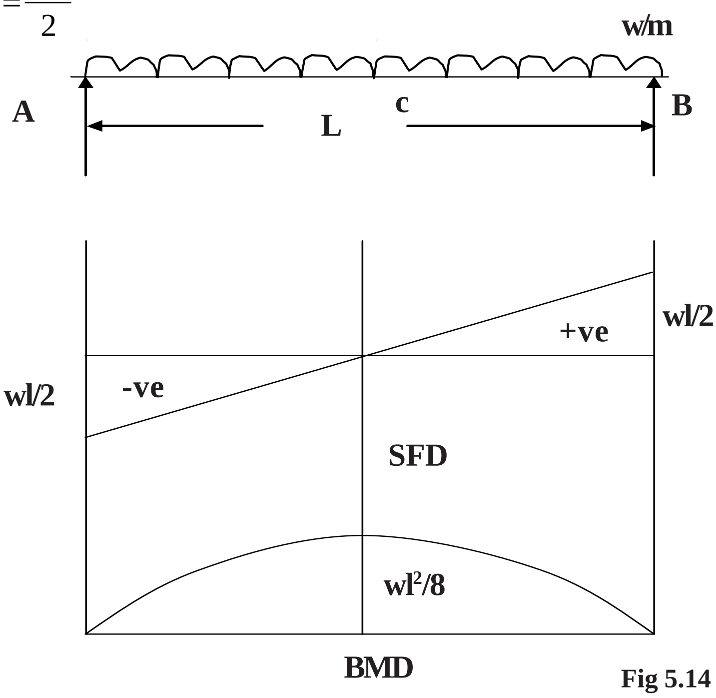


Fig 5.14

Shear force :

$$F_B = Shear\ force\ at\ B = +\frac{WL}{2}\left(Right\ upward + ve\right)$$

$$F_A = Shear\ force\ at\ A = \frac{WL}{2} - WL = -\frac{WL}{2}$$

$$F_C = Shear\ force\ at\ C\left(Mid\ point\right)$$

$$= +\frac{WL}{2} - \frac{WL}{2} = 0$$

Thus S.F. is equal to $\frac{WL}{2}$ at B and decreases uniformly by a stright line Law, to zero at the mid-point of the beam. It continues to decrease uniformly to $\frac{-WL}{2}$ at A.

Bending Moment:

$M_A = M_B = 0$ ($\because$ Simply supported ends)
M_C = Bending moment at mid point

$$= \frac{WL}{2} \times \frac{L}{2} - \frac{WL}{2} \times \frac{L}{2} \left(\frac{L}{2} \right)$$

$$= \frac{WL^2}{4} - \frac{WL^2}{8} = \frac{WL^2}{8} (Max.B.M.)$$

Example 5.6

A simply supported beam 6 m long is carrying a u.d.l. of 2 kn/m over a length of 3m from the right end. Draw SFD and BMD calculate the maximum B.M.

Solution:

Taking moments about A,

$$R_B \times 6 = 2 \times 3 \left(\frac{3}{2} + 3 \right)$$

$$R_B = \frac{27}{6} = 4.5KN$$

$$R_A = 2 \times 3 - 4.5 = 1.5KN$$

Shear force:

$$F_B = + R_B = + 4.5\,KN$$

$$F_C = + R_B - 2 \times 3 = 4.6 - 6 = -1.5Kn$$

$$F_A = - R_A = -1.5\,Kn (Left\,Upward - ve)$$

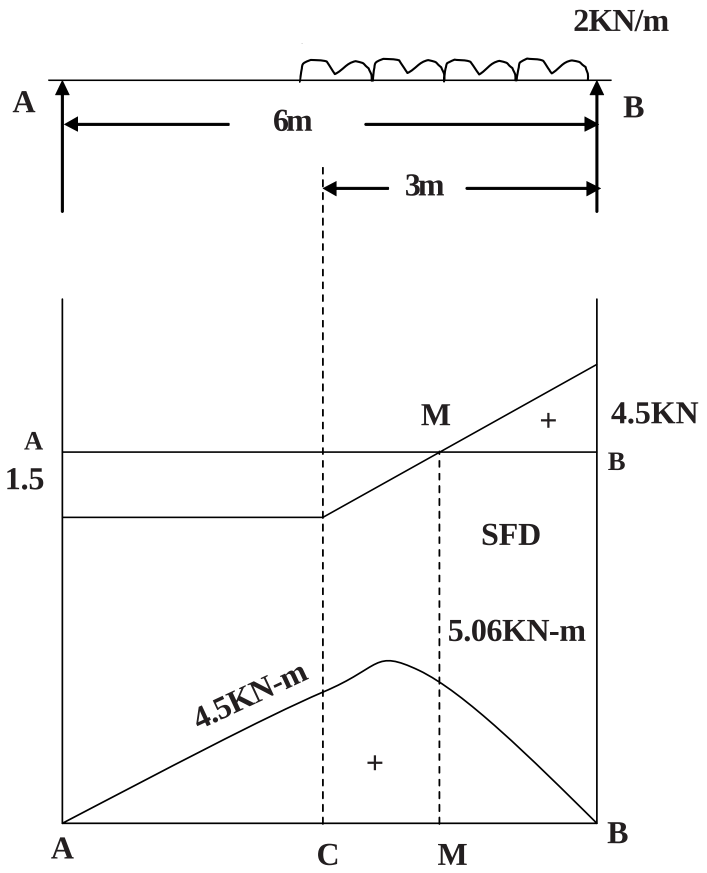


Fig. 5.15

Bending Moment :

$$M_A = M_B = 0$$

$$M_C = 4.5 \times 3 - 2 \times 3 \times \frac{3}{2} = 4.5\,KN - m$$

Maximum B.M. will occur at a point where S.F. changes sign. Let M be the point and x is the distance between C and M.

$$\therefore \qquad \frac{x}{1.5} = \frac{3 - x}{4.5}$$

$$4.5x = 4.5 - 1.5x$$

$$x = 0.75m$$

$$M_{\max} = 4.5 \times (3 - 0.75) - 2(3 - 0.75)\left(\frac{3 - 0.75}{2}\right)$$

$$= 5.06\,KN - m$$

Example 5.7

Draw SFD and BMD for the beam shown in Fig.5.16 also find maximum values of S.F., and B.M.

Solution :

Taking moments about A,

$$R_B \times 6 = 10 \times 4 \times \frac{4}{2} + 12 \times 2 + 8 \times 5$$

$$R_B = \frac{144}{6} = 24\ KN$$

$$R_A = (12 + 8 + 10 \times 4) - R_B$$

$$= 60 - 24 = 36\,KN$$

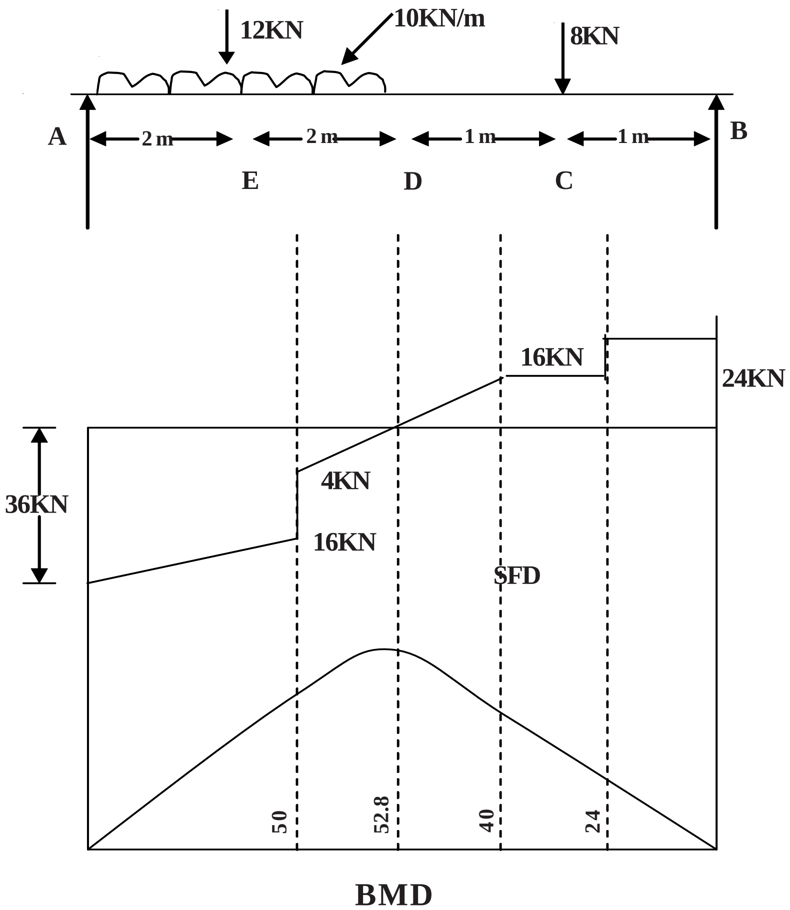


Fig. 5.16

Shear Force:

$$F_B = +R_B = +24\,KN$$
$$F_C = 24 - 8 = 16\,KN$$
$$F_D = 16\,KN$$
$$F_E\ (Right) = +24 - 8 - 10 \times 2 = -4\,KN$$
$$F_E\left(Left\right) = +24 - 8 - 10 \times 2 - 12 = -16\,KN$$
$$F_A = -R_A = -36\,KN$$

S.F. Changes sign in section ED. Let x be

the distance between A and the point where SF is zero.

$$Now,\ F_X = -36 + 12 + 10x = 0$$
$$\therefore x = \frac{24}{10} = 2.4\,m$$

Bending moment:

$$M_A = M_B = 0$$
$$M_C = 24 \times 1 = 24 KN - m$$
$$M_D = 24 \times 2 - 8 \times 1 = 40\,KN - m$$
$$M_E = 24 \times 4 - 8 \times 3 - 10 \times 2 \times \frac{2}{2} = 52\,KN - m$$

maximum bending moment will occur where S.F. changes sign.
At x = 2.4m, from left support,

$$M_X = +36 \times 2.4 - 12\left(2.4 - 2\right) - 10 \times 2.4 \times \frac{2.4}{2}$$
$$= 52.8\,KN - m$$

EXERCISE 5.2

1. A simply supported beam of span 4m has to carry a load of 10KN at the centre. Draw S.F.D. and B.M.D.

2. A horizontal beam, 12m long, simply supported at its ends is subjected to vertical point loads of 10KN, 20KN and 25 KN at 3m, 7m and 10m from the left support respectively. Draw the S.F. and B.M. diagrams indicating the values at salient points. (I.P.E. Sept. 95/March 2007)

3. A simply supported beam of 4m long is loaded with u.d.l. of 10KN/m for a length of 2m from left supplort. Draw S FD and BMD.

4. A simply supported beam of 6m long is loaded with u.d.l. of 20N/m for a length of 3m starting at 1.5 m from left support. Draw S.F.D. and B.M.D.

5. A simply supported beam 10m long is loaded with an u.d.l. of 150N/m on a length of 4m from the right hand support. It is also carrying a point load of 100N acting at 3m from left support. Draw S.F.D. and B.M.D.

5.10 S.F. and B.M. diagrams for over han`ging Beams

An overhanging beam is a beam, which overhangs from its support. It may overhang on one side only or on both sides.

Point of contraflexure :

An overhanging beam may develop both sagging and hogging bending moments. Hence, in an overhanging beam there will be a point, where the B.M. will change sign from negative to positive or vice versa. This point is called point of contraflexure.

Examples 5.8

A beam of 6m is supported at left end and at a distance of 1m from right end. It carries a point load of 20KN at a distance of 2m from left support. It also carries a load 10KN at free end, Draw SFD and B.M.D. for the beam.

Solution :

Taking moments about A,

$R_C \times 5 = 20 \times 2 + 10 \times 6$

$R_C = \frac{100}{5} = 20KN$

$R_A = (20 + 10) - 20 = 10KN$

Shear Force:

$F_D = -10\ KN$

$F_C = -10 + 20 = 10KN$

$F_B = -10 + 20 - 20 = -10KN$

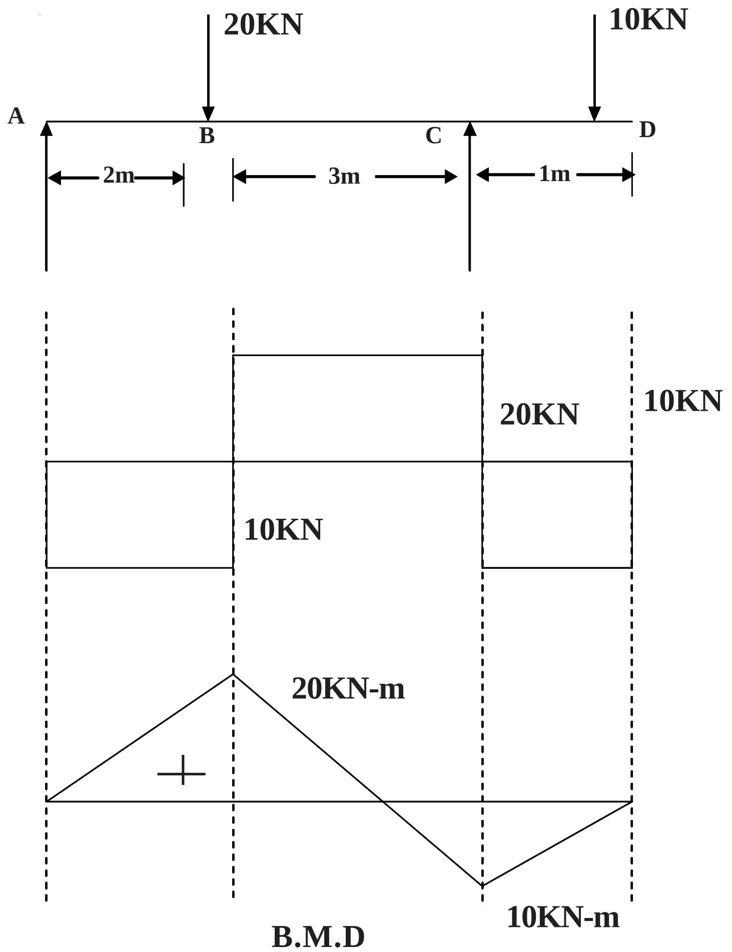


Fig. 5.17

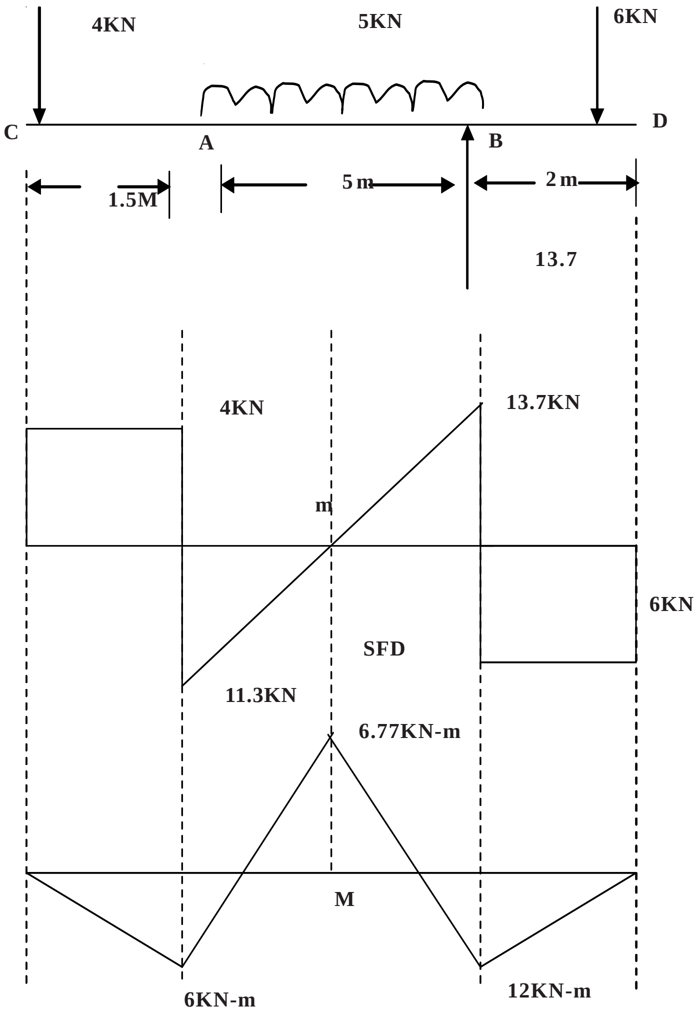


Fig. 5.18

B.M.D.

Bending moment:

$$M_D = 0$$
$$M_C = -10 \times 1 = -10 KN - m$$
$$M_B = -10 \times 4 + R_B \times 3 = -40 + 20 \times 3 = 20\, KN - m$$
$$M_A = -10 \times 6 + R_B \times 5 - 20 \times 2$$
$$= -60 + 20 \times 5 - 40 = 0$$

Point of contraflexure:

Let the point of contraflexture be at a distance x from c.

$$M_x = -10(1+x) + R_B\, x = 0$$
$$-10 + 10x = 0$$
$$\therefore x = 1m$$

Example 5.9

A beam 8.5m long, rests on supports 5m apart. Right hand side overhangs to 2m and left hand side overhangs to 1.5m. The beam is carrying loads as shown in fig.5.18. Draw SFD and BMD. Find the point of contraflexure.

Solution:

Taking moment about A,

$$R_B \times 5 + 4 \times 1.5$$
$$= 6 \times 7 + 5 \times 5 \times 2.5$$
$$R_B = \frac{98.5}{5} = 19.7KN$$
$$R_A = (5\times5 + 6 + 4) - 19.7$$
$$= 15.3KN$$

Shear force:

$$F_D = -6KN$$
$$F_B = -6 + R_B = -6 + 19.7 = 13.7\,KN$$
$$F_{A(Right)} = -6 + R_B + 5\times5 = -6 + 19.7 - 25 = -11.3KN$$
$$F_{A(left)} = -11.3 + R_A = -11.3 + 15.3 = +4KN$$
$$F_C = +4 - 4 = 0$$

Bending moment

$$M_D = M_C = 0$$
$$M_B = -6\times2 = -12\,KN - m$$
$$M_A = -4\times1.5 = -6Kn - m$$

Maximum bending moment, positive or negative, will occur at points where S.F. changes sign. Here it will occur at point A or M or B. Let x be the distance between B and M. We know,

$$\frac{x}{13.7} = \frac{5-x}{11.3}$$

$$\therefore \; x = 2.74m$$

$$\therefore M_m = -6(2+2.74) + 19.7 \times 2.74 - 5 \times 2.74 \times \frac{2.74}{2}$$

$$= +6.77\,KN - m$$

The maximum bending moment will occur at B

$$\therefore M_{\max} = -12\,KN - m$$

Point of contraflexure:

Let the point of contraflexure be at a distance x from B.

$$M_X = -6(2+x) + 19.7\,x - 5x\frac{x}{2} = 0$$

$$5x^2 - 22.6x + 12 = 0$$

$$\therefore \frac{+22.6 \pm \sqrt{(22.6)^2\; 4 \times 5 \times 12}}{2 \times 5} = 0.62m \; and \;\; 3.91m$$

EXERCISE 5.3

1. A beam of 6m is supported at left end and at a distance of 1m from right end. It carried a point load of 20KN at a distance of 2m from left support. It also carries a load of 10KN at a free end. Draw shear force and Bending moment diagrams. (I.P.E. March 96)

2. A beam of 8m is supported at a distance of 1m from left end and at a distance of 2 m from right end. It carries two point loads 4KN and 6KN at left and right ends respectively. It also carries a u.d.l of 5KN/m over the entire span. Draw SFD and BMD.

3. A beam of 6m long is simply supported at 1m from each end. It carries two point loads of 30 KN each at the two ends and 50KN at mid span. Draw SFD and BMD and find the points of contraflexure.

4. A beam of span 5m is supported at left end and at a distance of 2m from right end. It carries a point load of 15KN., at 1.5m from left end. It also carries a u.d.l. of 20 KN/m over entire span. Draw SFD and BMD. Locate points of contraflexure.

5.11 Relation between rate of loading, SF and BM.

Consider a beam carrying a uniformly distributed load of W per unit length. Now consider two sections AB and CD at a distance dx apart. The load acting between these two sections is equal to w.dx.

Let F = S.F. at AB
F + df = S.F. at CD
M = B.M. at AB
M + dm = B.M. at CD

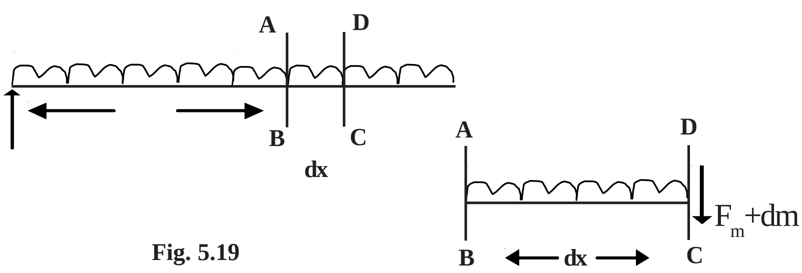


Fig. 5.19

Since the beam between the sections is in equilibrium, equating the unbalanced vertical forces,

F + W.dx = F + dF

$$\therefore \frac{dF}{dx} = w \text{..........................}(1)$$

Thus the rate of change of S.F. at any section is equal to the intensity of loading.

Similarly equating the moments in the section,

$$M - F.dx - \frac{w.dx^2}{2} = M + dM$$

$$dM = -F.dx \text{ (Neglecting the Square of small quantity } dx^2\text{)}$$

$$\frac{dM}{dx} = -F$$

Thus the rate of change of B.M. is equal to the S.F. at the section. For maximum B.M. the S.F. is zero.

SHORT ANSWER QUESTIONS

1. What are the types of beams?
2. What are the types of loads acting on beams?
3. Explain effective span and clear span of a beam.
4. Define shear force and bending moment.
5. St ate the values of max. S.F. ad Max. B.M. and their positions for a simply supported beam with u .d.l. over the span.
6. Define point of contraflexure.
7. A cantilever beam 3m long is loaded with a u.d.l. of 2 KN/m over its whole length. Draw SFD and BMD.
8. Draw and explain bending moment diagram of a simply supported beam carrying point load at the middle of the span.
9. Explain briefly the relationship between shear force and Bending moment at a section.

6
GRAPHIC STATICS

6.1. Introduction:

In Mechanics, many problems are solved by two methods.

1. Analytical method 2. Graphical Method.

In some problems the solutions are very cumber some. These problems can be solved very easily by graphical methods. It is a simple method used to find forces in the members of a truss, to find the c.g. of plane figures, to draw SFD and BMD for loaded beams. In this method the accuracy of the result depends on the care taken regarding the constructions.

6.2 Bow's Notation

The loads on beams, forces and different members of a frame are represented by two capital letters, placed in the spaces on either side of them either clockwise (or) anti-clockwise. This is called as Bow's notation. Fig.6.1 shows Bows notation for a system of forces.

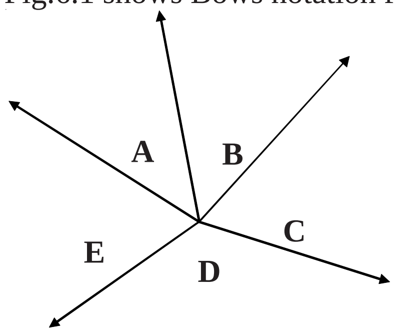


Fig.6.1

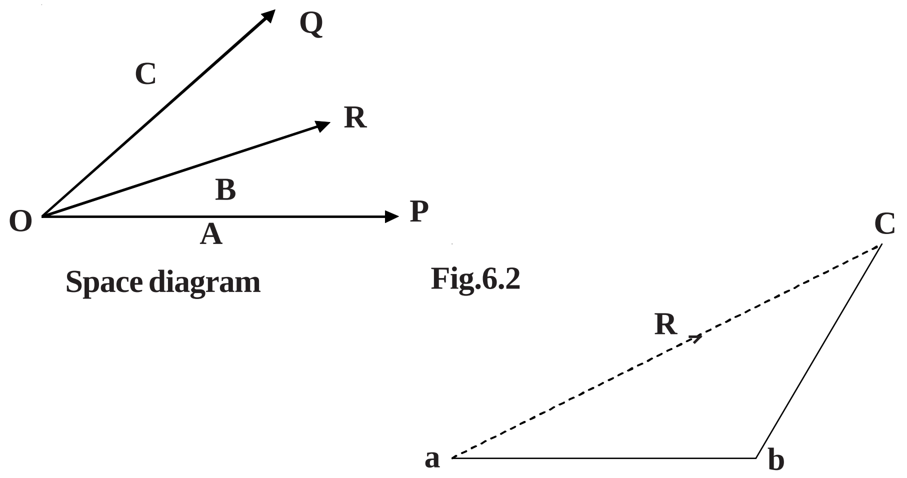
6.3 Space and vector diagrams:

Space diagram is simply a scaled diagram showing the point of application and line of action of the forces. This diagram represents the exact location of the force (or) load including the support reactions drawn to any suitable scale.

After drawing the space diagram, the next step is to draw the vector diagram. This diagram shows the magnitude, direction and sense of each force of the space diagram.

6.4 Resultant of system of forces graphically:

Let P and Q are two co-planar concurrent forces acting on a body at O. The resultant of the two forces can be found out graphically by drawing space diagram and vector diagram.



Space diagram

Fig.6.2

FIG. 6.2

Vector diagram

Procedure:

1. Draw the space diagram and name the forces according to Bow's notation.
2. Draw a line ab to represent the force AB (force p) to suitable scale.
3. From b, draw bc parallel to the force BC (force Q) to the same scale.
4. Join ac which gives the magnitude of the resultant R.
5. In the space diagram draw a line through O and parallel to ac.
6. Measure the angle with reference to any force P or Q which gives direction of the resultant.

Example 6.1

Find the resultant of two forces 150N and 250 N act at a point at an angle of 60^0

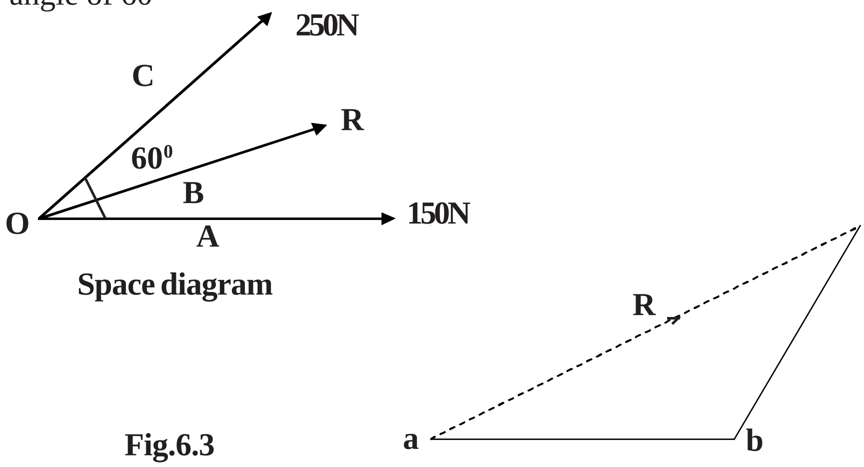


Fig.6.3

Vector diagram

Solution:

1. Draw the space diagram to show the two forces and name the forces according to Bow's notation.
2. Draw a line ab to represent force 150 N to suitable scale (for example 1 mm = 5N)
3. From b, draw bc parallel to the force 250N to the same scale.
4. Join ac which gives the magnitude of the resultant R.
5. In the space diagram draw a line through O and parallel to ac.
6. Measure the angle with reference to force 150N which gives the direction of the resultant

Result: 1) Length of ac = 70 MM
2) Resultant force (R) = 350N
3) Direction of resultant with reference to 150N force = $38^0 12^1$

Example 6.2

Find the resultant force for the system of forces 100, 150, 200, 250 and 300N mutually acting at 30^0, 60^0, 120^0 and 200^0

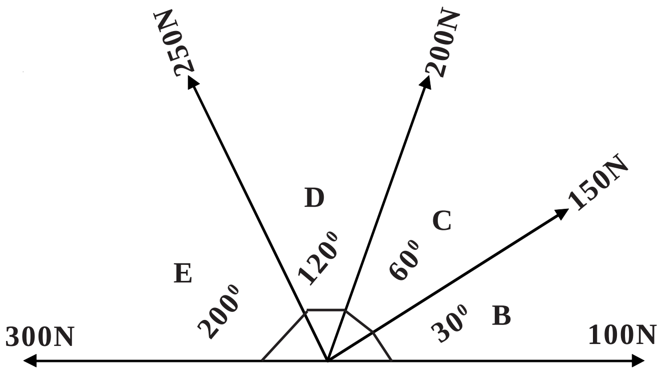
Solution:

1. Draw the space diagram to represent the forces and name them as poer bow's notation.

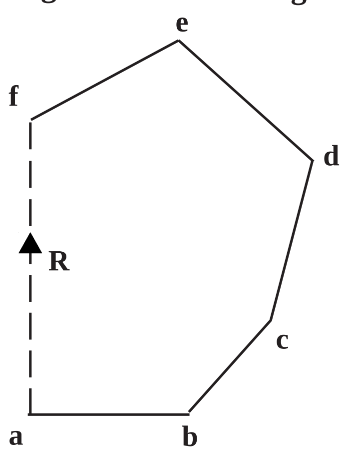
2. Draw a line ab representing the force AB to a suitable scale say 1 mm = 5N
3. At b draw a line bc parallel to the force BC to the same scale, at c draw cd parallel to the force CD, etc.
4. Join af, which is the resultant. In the space diagram draw a line parallel to af and passing through 'o'. Measure the angle of the resultant with reference to any force.

Result :

1. The length of the line af = 31 mm
2. The magnitude of the resultant = 155N
3. Direction of the resultant =80^0 with 100N force.



Space diagram **Fig.6.4**



Vector diagram

Example 6.3

A wheel has five equispaced spokes. The force acting in three spokes are 6KN, 7KN and 5KN. Find the forces acting in the other two spokes for the wheel to be in equilibrium.

Solution :

The five spokes are equispaced, so the angle between adjoining spokes will be $360^0/5 = 72^{0.}$

1. Draw the space diagram with angle between adjoining spokes as 72^0. Represent the forces by Bow's notation.

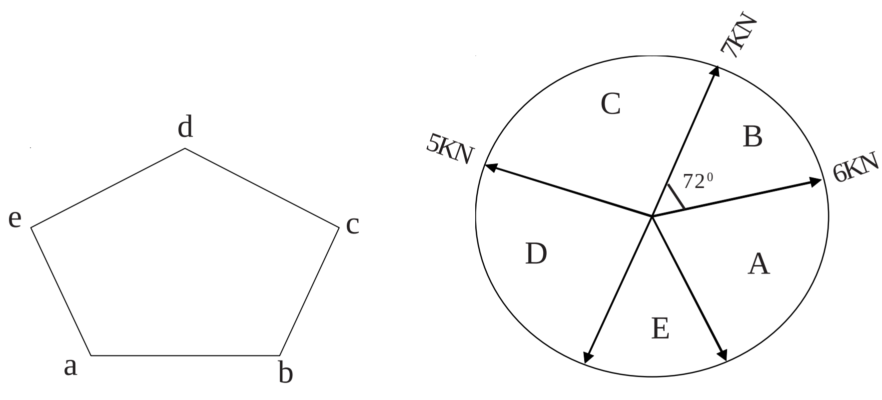
2. Draw vector diagram abcd to represent the forces 6KN, 7KN, 5KN to suitable scale say 1CM=1 KN.

3. Draw line 'de' parallel to DE and 'ea' parallel to EA to intersect at e.

4. Measure the lengths de and ea which give the magnitudes of forces in spokes DE and EA.

Result :

1) Force in spoke DE =7.2 KN
2) Force in spoke EA =5.6 KN



Vector diagram(Scale 1cm=1KN) **Space diagram**

Fig. 6.5

Example 6.4

Four forces 100N, 200N, 250N, 350N act along the sides of a square of side 1.5m. Find graphically the resultant

Solution :

1. Draw the space diagram and name the forces as per Bow's notation.

2. Draw the vector diagram abcde parallel to forces. Select pole 'o' join oa, ob, oc, od and oe.

3. Draw lines parallel to oa, ob, oc, od and oe cutting the lines of forces AB, BC, CD and DE at 1,2,3 and 4 in space diagram.

4. Extend the first and last sides, draw parallel lines to oa and oe in space diagram to meet at a point5.

5. At 5 draw a parallel line to ae and measure the perpemdicular distance between this line and any corner of square.

Result :

1. The magnitude of the resultant = 212N

2. Direction of the resultant = 45^0 to horizontal

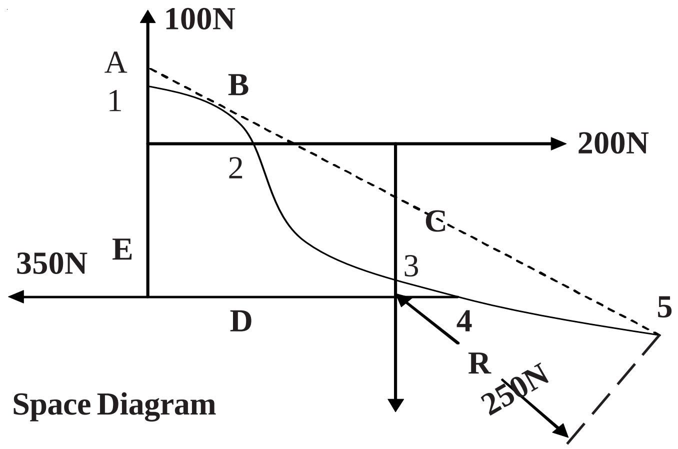
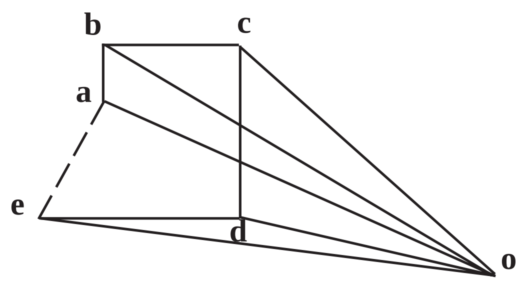


Fig. 6.6



Vector Diagram

Example 6.5

A beam of 4m long is simply supported at the ends. It is loaded with three like parallel forces 2KN, 3KN and 4KN act at 1m, 2m and 3m respectively from the left end. Find the reactions at the ends.

Solution.

1. Draw the space diagram of the beam using suitable scale 10mm=1m. Mark the point loads.

2. Draw a vector diagram to a suitable scale of 10 mm= 1KN. Name the points.

3. Select a pole o and join oa, ob, oc,od and oe.

4. Draw a funicular polygon 12345 join 1 and 5.

5. In the vector diagram draw a line parallel to 1-5 line and passing through o. It cuts the line at f. Measure df and fa which gives reactions of supports.

Results.

1 The reaction at left support = 4KN

2 The reaction at right support =5KN

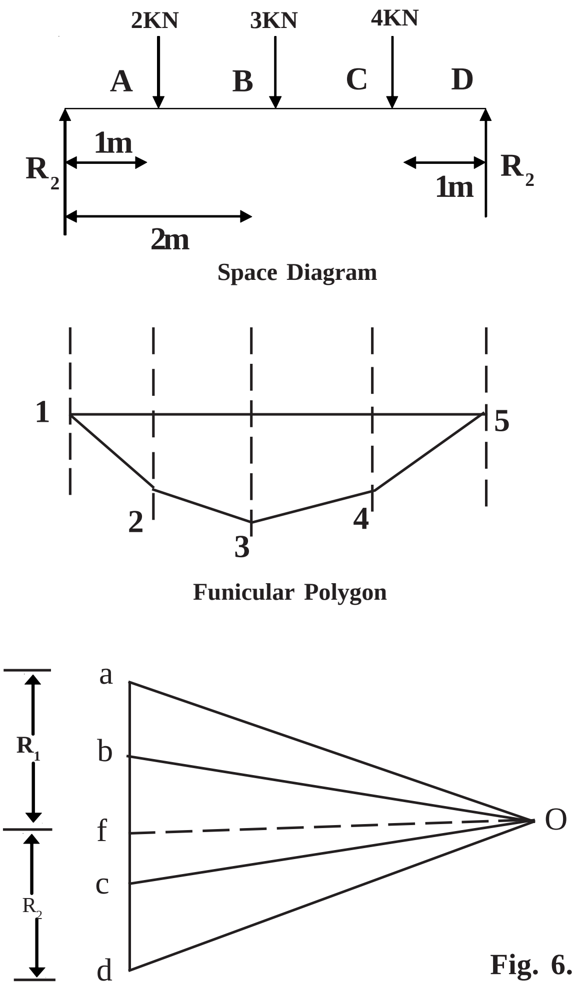


Fig. 6.7

Vector Diagram

EXERCISE 6.1

1. Find the resultant force of system of forces 300N, 600N, 400N and 200N making angles of 30^0, 120^0, 225^0 and 330^0 respctively with a horizontal line. Use graphical method.

2 A particle is acted upon by three forces of 50N, 100N and 120Nalong the three sides of an equilateral triangle. Find graphically the magnitude and direction of the resultant.

3 Find forces 8KN, 4KN, 6KN and 10KN act at the corner of a regular pentagon of side 1m. The direction of each pull is along the bisector of the angle at the corner. Determine graphically the magnitude and direction of the resultant.

4 A simply supported beam of span 6m is supported at its ends. It carries point loads of 30KN, 20KN, 30KN and 20KN at distances of 1m, 2m, 3m and 4m respectively.

5 A simply supported beam AB of span 8m is loaded with point loads of 10N, 15N, 30N at distances of 2, 4 and 6m from right support. Determine graphically the support reactions.

6.5 Graphical method of finding the centroid

The following procedure is used to find the centroid of plane areas.

1. Divide the given area into number of regular figures.

2. Draw parallel lines through centroid of each part in horizontal and vertical directions and name the spaces as per Bow's notation.

3. Draw vector diagram and find the resultant in two directions. The point of intersection of two resultant lines is the centroid of the area.

Example 6.6

Determine graphically the centroid of a T-section 180 mm X 20 mm X 20mm

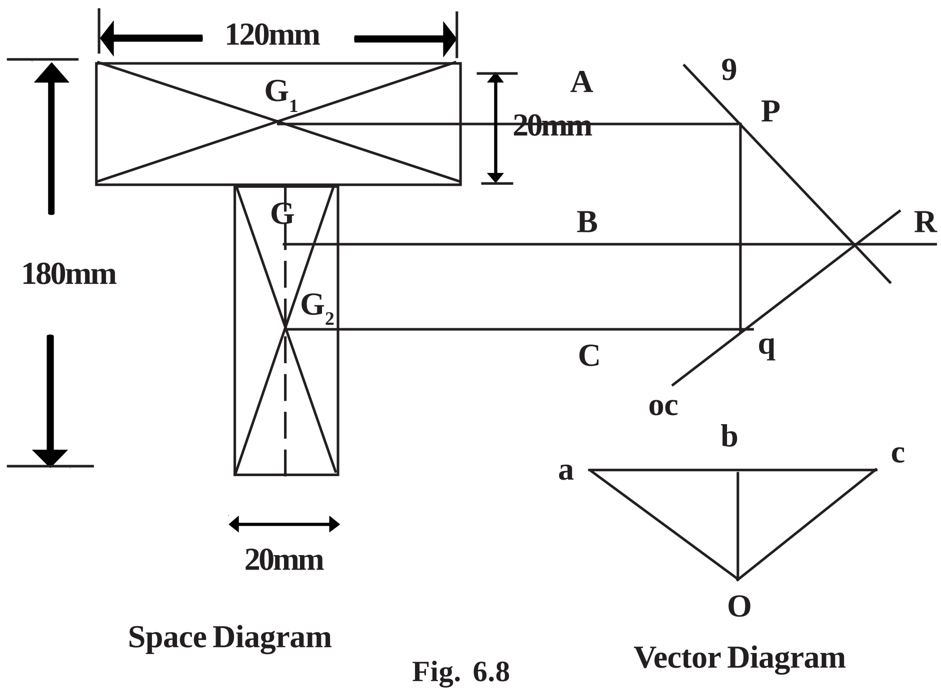


Fig. 6.8

1 The figure is symmetrical about y-axis so the cenroid lies on this line. Draw a space diagram to a scale 1:3.

2 Divide the area into two rectangles and locate the centroids by joining the diagonals

3 Draw horizontal lines from centroids and name the speces as per Bow's notation.

4 Draw a vector diagram to a suitable scale 1 mm = 100 mm^2 . Draw lines ab and bc to represent the areas of rectangle. Select a pole O and join oa, ob and oc.

5 Draw lines parallel to oa, ob citting AB and BC at p and q respectively. At point q draw a line parallel to oc. The extensions of lines oa and oc meet at a point R.

6 Draw a parallel line from R meeting the vertical line of space diagram at G. Measure the distance G from top or bottom of T- section. The centroid G is at a distance of 120 mm from bottom.

6.6 Graphical method of drawing S F D and B M D

The following procedure is adopted to draw S F D and B M D graphically for the loaded beams :

1 Name the spaces as per Bow's notation. Project the lines of action of forces in the space diagram.

2 Draw vector diagram, select a point at suitable distance (say 50 mm) and join the points. This is polar diagram.

3 Project the horizontal lines from vector diagram. Join the points of intersections. This is S F D.

4 Construct a funicular polygon and complete the polygon by drawing closing line. This is B.M.D

Example 6.7

A cantilever beam of span 6 m is loaded with point loads of 4 KN and 2 KN at a distance of 2 and 5 m from fixed support. Graphically draw S F D and B M D.

Solution.

1 Draw the space diagram to a suitable scale. Name the spaces as per Bow's notation. (Fig. 6.9)

2 Draw the vector diagram abc, select a pole 'o' and join oa, ob and oc.

3 Draw horizontal lines through points a,b and c, join the points of intersections. This is S.F D.

4 Measure the maximum ordinate in SFD and multiply with vector scale gives the maximum shear force.

5 Draw the corresponding funicular polygon. This is B.M.D. Measure the maximum ordinate in B.M.D and multiply with bending moment scale gives the maximum bending moment.

Scale of B.M.D

1 mm = space scale X Vector scale X polar length
=100 X0.1X 50=500 KN-mm
=0.5 KN –m

Example 6.8

A simply supported beam of span 4m is subjected to a u.d.l of 2 KN/m over the whole span. Draw S.F.D and B.M.D using graphical method.

Solution.

1) Draw the space diagram to a suitable scale. The U.D.L. on the beam should be converted into equivalent point loads. For this, divide the span into 8 equal parts each 0.5m long. Load in each part $\left(2\times\frac{4}{8}=1KN\right)$ is a point load acting at the mid point. Name the spaces as per Bow's notation.

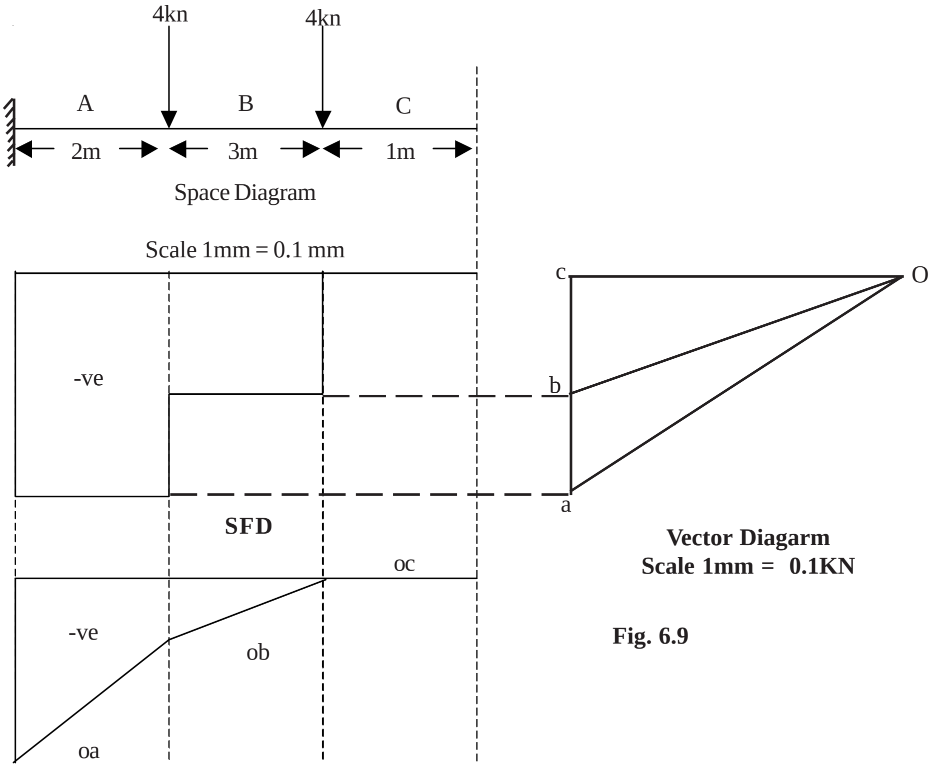
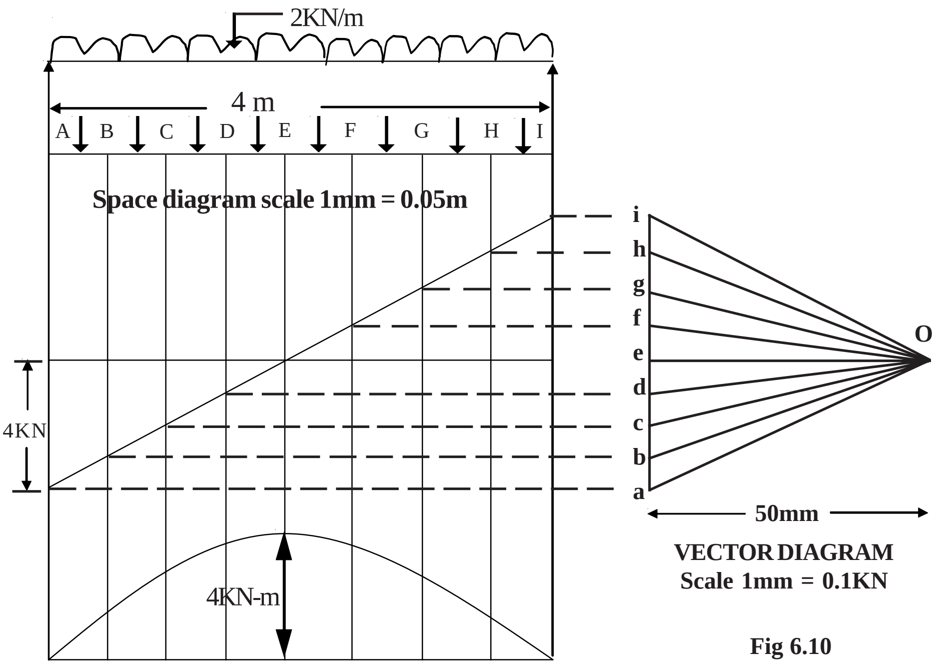


Fig. 6.9

BMD

Result : 1) Maximum shear force = 6kN
2) Maximum bending moment = 18KN-m



BMD Scale 1mm = 0.25 KN-m

Fig 6.10

2) Draw the vector diagram. Select a pole 'O' at a distance of 50mm and join the points.

3) Draw funicular polygon and the line drawn parallel to the closing line of the funicular polygon from the pole cuts the load line at 'e'.

4) Draw a line parallel to the span through 'e'. Project the points a,b,b,c,d...in the respective spaces A,B,C,D,.... to get the S.F.D. In projecting the S.F.D. a series of vertical steps would be obtained under each point load.

5) The B.M.D. Can be drawn by joining the verticals of the funicular Polygon by a smooth curve.

EXERCISE 6.2

1 Find the centroid of I-Section with top flange – 80 X 20 mm, web – 100 X 20 mm, bottom flange 100X 20 mm. Use graphical method.

2 Graphically find the centroid of L-section 150 mm X 100 mm X 10 mm

3 A cantilever of 6 m span is carrying point loads of 15 KN, 20 KN and 30 KN from fixed end at a distance of 2,4 and 6 m. Draw S.F.D and B.M.D graphically

4 A cantilever beam of span 5 m is subjected to a u d l 12 KN /m throughout the span Draw S.F.D and B.M.D graphically. Find maximum S.F and B.M values.

5 A simply supported beam of span 5 m is carrying point loads of 2 KN, 4 KN, 6 KN at points 1, 2.5, 4m from left end. Draw S.F.D and B.M.D.

6 A simply supported beam of span 6m is subjected to an u.d.l of 15 KN/m over the entire span. Draw S.F.D and B.M.D graphically. Find maximum SF and B.M